JOURNAL OF HOUSING ECONOMICS **2**, 17–36 (1992)

# Optimal Mortgage Design When Transaction Costs Constrain Mobility*

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Received May 26, 1992

In the presence of transaction costs, purchasers of housing services plan multiyear stays in specific locations by optimizing with regard to present and future preferences, incomes, and prices. In equilibrium, the sum over time of the weighted differences between the marginal rate of substitution and price ratio equals zero. This resembles a public good equilibrium, and the "Lindahl Solution" would call for borrowers to use variable payment mortgages. In this paper we fully develop the concept of an optimal mortgage by using a dynamic model in which transaction costs contrain mobility. We find that variable payment mortgages benefit consumers through reduced transaction costs and benefit the lender through a larger housing purchase earlier in life. A simulation is conducted to validate and quantify these theoretical findings with measures of equivalent and compensating variations. Impediments to Lindahl-type variable payment mortgages are discussed, and solutions are proposed. © 1992 Academic Press, Inc.

## 1. INTRODUCTION

Two of the more widely studied topics in the economics of housing have been housing demand and the choice of a mortgage instrument. The relationship between these topics is examined here.

Housing demand has been largely examined within static models in which tenure choice (own or rent) and the decision to move is considered jointly with the consumer's demand for housing services. There has been little consideration to determining a consumer's multiperiod housing demand. A dynamic analysis that accounts for transaction costs is necessary for a better understanding of observed housing consumption changes.

* We are grateful to Ralph Braid, James Follain, Masahiro Kawai, Henry Pollakowski, Chester Spatt, and an anonymous referee for their insightful comments. We also thank seminar participants at Wayne State, the University of British Columbia, and the January 1992 AREUEA Meetings. The usual disclaimer applies.



1051-1377/92 $5.00

In parallel work (Goodman, 1992), dynamic equilibrium with regard to housing demand occurs where the sum, over time, of the weighted differences between the marginal rate of substitution (between housing and the other consumption good) and the price ratio equals zero. This resembles a public good equilibrium, and the "Lindahl Solution" would call for borrowers to use variable payment mortgages (VPMs). In this paper we fully develop the concept of an optimal mortgage by using a dynamic model in which transaction costs constrain mobility. We find that mortgage instruments of this type offer potential benefits to both borrowers and lenders.

Section 2 contains a review of previous literature on mortgage financing and its effect on housing demand. In this review, literature relating to adjustable rate mortgages is also examined. Many of the issues and questions raised in this literature are addressed in our work. Section 3 examines housing demand in the presence of transaction costs. Section 4 introduces borrowing into the model, and Section 5 provides an analytical solution to a two-period model in which moving is endogenous. Section 6 links borrowers' and lenders' decisions. Section 7 presents a simulation as well as consumer and seller surplus calculations. Section 8 contains the conclusion and possible extensions of this work.

## 2. MORTGAGE FINANCING, HOUSING DEMAND, AND ADJUSTABLE RATE MORTGAGES

An extensive examination of mortgage financing and its relationship to housing demand and tenure decisions has been conducted by Plaut. He presents his basic model in Plaut (1984), with extensions in Plaut (1987). In Plaut's model a household lives from 0 to $T$ and knows its lifetime income in advance. The price of housing and its depreciation are exogenous. The household must choose a lifetime consumption pattern among housing, an aggregate consumption good, and a bequest. In the first stage of the life cycle a household rents housing in order to accumulate savings to make a down payment on a house. During this period, rental housing can be instantaneously adjusted. A household decides to purchase a house at time $t_1$ with accumulated savings and a mortgage. The mortgage is paid back at a time $t_2$ which is halfway between $t_1$ and $T$. The amortization of the mortgage $M$ is exogenously given. The real interest rate on the mortgage $r$ is an increasing function of the loan to value ratio.

In Plaut's model, households are motivated to consume less housing (including rental) to boost the size of down payment. He finds that the timing of tenure transition is delayed by higher mortgage interest rates, by higher housing rental rates, and by shorter periods of mortgage amortization.



Plaut (1985) uses the same model to show that an increase in the leverage sensitivity of the mortgage interest rate leads to flatter housing gradients and greater suburbanization. Most relevant to our work, Plaut (1986) again presents his basic model but allows households to decide on the amortization schedule of mortgage $M$. With $\alpha$ a choice variable, a housing consumer makes payment $Z_1 = \alpha M(1 + r)$ in period $t_1$, and payment $Z_2 = (1 - \alpha)M(1 + r)^2$ in period $t_2$. Plaut finds the first partial derivative of the expected utility with regard to $\alpha$ to be ambiguous in sign. The optimal value of $\alpha$ is positively related to income in period $t_1$, the degree of risk of the nonhouse savings asset, the mortgage interest rate, and the consumer's being restricted to a mortgage less than the preferred amount. The optimal $\alpha$ is negatively affected by the expected return on housing and the expected return on nonhouse saving.

Plaut also shows housing demand to be negatively related to $\alpha$ if the covariance between the price of housing and the consumption of the alternate consumption good is negative. A higher amortization schedule reduces hedging possibilities and leads to a reduction in housing. Plaut shows that it is possible to derive optimal mortgage repayment schedules for households; he also demonstrates how these schedules are affected by different variables. Most importantly, he shows that a mortgage's scheduled amortization can influence housing demand.

Previous work relating mortgage choice to housing demand in an inflationary and liquidity constrained environment is also relevant. High expected inflation drives up the nominal interest rate in a fixed rate mortgage and "tilts" the real value of fixed payments toward the early years of the loan. This tilt can reduce housing demand if consumers are constrained by imperfect capital markets. A graduated payment mortgage (GPM), which permits a reduction in initial mortgage payments in exchange for an increase in later payments, has been proposed as a solution to inflation tilt. Follain (1990) discusses this and other related issues.

Alm and Follain (1982), by use of a simulation, assess the effect of mortgage choice in an inflationary and liquidity-constrained environment on the timing and quantity of individual housing decisions. Inflation both stimulates housing demand through a reduction in the after-tax cost of housing and depresses housing demand by making liquidity constraints more severe. They find that inflation below 10% increases housing demand, while greater inflation decreases it. Rising prices also encourage homeownership sooner. In an inflationary environment, GPMs can substantially increase housing demand. Alm and Follain find that households are willing to pay over half a point more for a GPM (than for a standard mortgage instrument) when the graduation rate approximates the inflation rate.

Schwab (1982) confirms the findings of Alm and Follain. He proposes a price-level-adjusted mortgage where consumers always make constant real payments. In his simulation, the switch from a fixed payment mortgage to a price-level-adjusted mortgage would increase housing demand by 8% and would cause a utility gain. We do not address the effects of inflation, but we do examine the effects of constraints imposed by mortgage instruments more broadly.

In response to the 1980s' relatively high fixed-rate mortgages (FRMs) and the obvious influence that mortgage repayment schedules have on housing demand, adjustable rate mortgages (ARMs) became popular. Brueckner and Follain (1988) examine the rise in the ARMs' popularity and find that it can be attributed both to the FRM interest rate and to the interest rate differential between FRMs and ARMs. Households with high incomes and a greater degree of intermetropolitan mobility also prefer ARMs.

In an extension of this work, Brueckner and Follain (1989) estimate the effect of ARM/FRM choice on housing demand with a two-stage method. The first stage is the same as their 1988 work, where a probit equation is used to estimate ARM/FRM choice. The second stage is the estimation of a housing demand equation. Simulations show ARM demand to be less sensitive than FRM demand to an increase in interest rates. Mobile ARM borrowers also demand more housing than sedentary ARM borrowers. At typical interest rates, a given individual demands more housing as an ARM borrower.

Brueckner and Follain conclude that the presence of ARMs stimulates and stabilizes the housing market. Our research suggests an optimal mortgage instrument that can further improve on the standard adjustable rate mortgage.

## 3. HOUSING DEMAND WITH TRANSACTIONS COSTS

The economics of housing demand have evolved substantially over the past two decades. Major improvements in the modeling of price and income terms, the development of discrete choice models, and the availability of large scale data bases have led to substantially improved models. Goodman (1989) discusses some of these changes in more detail.

The models, and particularly the empirical estimates, have remained static in nature. Households are viewed at one point in time. If they move during that period, they are often considered to be "closer to equilibrium," and their housing demand is estimated jointly with the decision to move. If they do not move, then their tenure choice (own or rent) is often estimated jointly with housing demand. This is done under the premise that they are "out of equilibrium" on the demand side, but that this disequilibrium is not severe enough to overcome the sizable transactions costs that moving entails.



Such analyses are at variance with the anecdotal wisdom that suggests:

—Consumers have good foresight as to future incomes and prices.
—Consumers are fairly well informed about the transactions costs of changing housing locations.

This study develops a model that explicitly treats transactions costs in the consumer optimization problem in a dynamic setting. Consider a consumer optimizing over $\tau$ periods. The transactions cost of moving each period is $m$. Ignoring saving and/or borrowing, if the consumer, at Time 0, is planning over the future, and moves each period, the sum of the discounted future utilities (using $D = 1 + \delta$, where $\delta$ is the rate of time preference) is

$$U^* = \sum_{t=1}^{t=\tau} D^{1-t} U^t(h_t^*, c_t^*), \tag{1}$$

where the budget constraint each period is

$$y_t = p_t h_t + c_t + m. \tag{2}$$

The consumer recognizes that incurring costs $m$ each period may decrease $U^*$ below the value if the household did not move. Suppose she solves a separate optimization in which she stays in the same unit in Periods 1 and 2, but moves in every remaining period, leading to $U^{**}$:

$$U^{**} = \sum_{t=1}^{t=2} D^{1-t} U^t(\bar{h}^{**}, c_t^{**}) + \sum_{t=3}^{t=\tau} D^{1-t} U^t(h_t^{**}, c_t^{**}), \tag{3}$$

where the budget constraints in Periods 1 and 2 are

$$y_1 = p_1 \bar{h} + c_1 + m, \tag{4}$$

$$y_2 = p_2 \bar{h} + c_2, \tag{5}$$

and for all subsequent times $t$

$$y_t = p_t h_t + c_t + m. \tag{6}$$

If $U^{**} > U^*$, then the second regime dominates the first, and she will stay in the home for the first two periods and then move every remaining period.

It is important to characterize the nature of the equilibrium that is generated during each stay. For convenience, assume that the consumer is evaluating the level of utility generated by a stay beginning in Period 1 and lasting $\tau$ periods. Again ignoring saving and/or borrowing, optimize

$$J^\tau = \sum_{t=1}^{t=\tau} D^{1-t}U^t(h, c_t) + \lambda_1(y_1 - p_1h - c_1 - m) + \sum_{t=2}^{t=\tau} \lambda_t(y_t - p_th - c_t). \tag{7}$$

The variable $\lambda_t$ signifies the marginal utility of income. The first-order conditions, optimizing with respect to $h$ and $c_t$ within that stay, are

$$\partial J^\tau/\partial c_t = D^{1-t}U^t_{ct} - \lambda_t = 0, \qquad \text{for all } t \tag{8}$$

and

$$\partial J^\tau/\partial h = \sum_{t=1}^{t=\tau} D^{1-t}U^t_h - \lambda_t p_t = 0. \tag{9}$$

Substituting, and rearranging, leads to the optimum condition

$$\sum_{t=1}^{t=\tau} D^{1-t}U^t_c[(U^t_h/U^t_c) - p_t] = 0. \tag{10}$$

The term $D^{1-t}U^t_c$ is the marginal utility of income in time $t$. Here, the individual who consumes housing over time is solving a problem similar to the socially optimal level of a *public good*. Instead of the government's optimizing across individuals for a specific time period, the housing consumer optimizes across time periods, subject to the constraint that housing consumption remains constant.[1] Equation (10) can be rewritten as

$$\sum_{t=1}^{t=\tau} MU^t_{inc}[MRS_t - p_t] = 0. \tag{11}$$

An alternative formulation of this model would have the consumer purchase housing at the same price as the initial period, or $p_1$.[2] For landlords to accept this arrangement, they must be able to maintain the present value of the rents. Consider a two-period model. If the consumer

[1] If the price ratio $p_t$ is constant over time, then the analogy to the public good is tightened.
[2] We are grateful to the anonymous referee for proposing this formulation.



pays $p_1$ each period, the moving cost would be reformulated, where $r$ is the interest rate:

$$m_1 = m + [p_1 + p_2/(1 + r)]h - [p_1 + p_1/(1 + r)]h. \tag{12}$$

Thus, if $p_1$ is less (greater) than $p_2$, the consumer's down payment (moving cost plus price difference) increases (decreases). Optimizing, as before, yields

$$MU^1_{inc}[MRS_1 - (p_1r + p_2)/(1 + r)] + MU^2_{inc}[MRS_2 - p_2] = 0. \tag{13}$$

Since we concentrate on models with capital markets in our further analyses, we do not pursue this alternative formulation further.

## 4. BORROWING IN THE MODEL

The opportunity to borrow allows the individual to spread her payment opportunities over the duration of the stay. To avoid the treatment of either capital gains or bequests, assume that she signs a contract to purchase a house for a $\tau$-period stay.[3] The asset owner is paid a lump-sum payment on the first day of Period 1 by the lender, and the lender holds the house as collateral. The consumer pays both transactions cost $m$ and payment $z_1$ the first day of Period 1. Thereafter, the consumer pays $z_t$ the first day of each Period $t$. The consumer vacates the dwelling at the end of the $\tau$th period, having accumulated no equity. Consumer default on the loan gives the lender the right to evict the borrower and to collect housing service payments for the remainder of the term of the loan.[4]

Consider first a mortgage with fixed payment $z$. The consumer buys housing service bundle $h$ for price $p^*h$ and repays the loan in equal installments $z$:

$$J = \sum_{t=1}^{t=\tau} D^{1-t}U^t(h, c_t) + \lambda_1(y_1 - z - c_1 - m) + \sum_{t=2}^{t=\tau} \lambda_t(y_t - z - c_t) + \mu(Fz - p^*h), \tag{14}$$

letting $p^* = \sum_{t=1}^{t=\tau} p_t(1 + r)^{1-t}$, and $F = \sum_{t=1}^{t=\tau} (1 = r)^{1-t}$.

[3] The addition of capital gains or bequests would simply change the terminal value of the dwelling unit from zero to some arbitrary constant, and thus would not change the analysis.
[4] As formulated, our model does not distinguish between buying and renting, and looks only at the optimal mortgage instrument to fund either. We could readily permit differential moving costs depending on the tenure considered.

Optimizing with respect to $c_t$ gives condition (8). Optimizing with respect to $h$ and $z$ yields

$$\partial J/\partial h = \sum_{t=1}^{t=T} D^{1-t} U_h^t - \mu p^* = 0 \tag{15}$$

and

$$\partial J/\partial z = \sum_{t=1}^{t=T} - \lambda_t + \mu F = 0. \tag{16}$$

Substituting for $\lambda_t$ and $\mu$ leads to

$$\sum_{t=1}^{t=T} MU_{inc}^t [(MRS)_t - (p^*/F)] = 0. \tag{17}$$

In contrast to the earlier analysis, the consumer optimizes over the entire stay with respect to a discounted *average* price ratio $p^*$ rather than the marginal price ratio $p_t$ in each period.[5] This results in some inefficiency in allocating consumer resources. However, the ability to save and/or borrow allows the marginal utility of income $\lambda_t$ in each period to converge.[6]

Regardless of a consumer's marginal utility of income in period $t$, the standard fixed-rate fixed-payment mortgage requires that payments be equal across all time periods. Consider instead the opportunity to vary payments subject only to the constraint that the discounted value of the variable-payment scheme equals the discounted value of the housing payments:[7]

[5] Recalling that the prices of the numeraire good are set at 1 in this example, if the numeraire prices for each period were $k_t$, then Eq. (17) would be rewritten as

$$\sum_{t=1}^{t=T} MU_{inc}^t [(MRS)_t - (p^*/k_t F)] = 0.$$

[6] With a FPM, the marginal utilities of consumption and housing adjust such that

$$(\Sigma D^{1-t} U_c^t)/(\Sigma D^{1-t} U_h^t) = F/p^*.$$

[7] Lenders might choose to place nonnegativity constraints on $z_t$, although mathematically it is not necessary to do so. It is also likely that an appropriate penalty could be derived that would allow lenders, in certain periods, to make payments to borrowers. We discuss this below.



$$J = \sum_{t=1}^{t=T} D^{1-t} U^t(h, c_t) + \lambda_1 (y_1 - z_1 - c_1 - m)$$
$$+ \sum_{t=2}^{t=T} \lambda_t (y_t - z_t - c_t) + \mu \left\{ \sum z_t (1 + r)^{1-t} - \left[ \sum p_t (1 + r)^{1-t} \right] h \right\}. \tag{18}$$

Optimizing with respect to $h$, we obtain

$$\partial J/\partial h = \sum_{t=1}^{t=T} D^{1-t} U_h^t - \mu \sum_{t=1}^{t=T} (p_t (1 + r)^{1-t}) = 0. \tag{19}$$

Optimizing with respect to $c_t$ and $z_t$, we obtain

$$\mu = \lambda_1 = \lambda_t (1 + r)^{t-1}, \text{ for all } t > 1. \tag{20}$$

Substituting into (19) yields

$$(U_h^1 - \lambda_1 p_1) + (D^{-1} U_h^2 - \lambda_2 p_2) + (D^{-2} U_h^3 - \lambda_3 p_3) + \cdots = 0,$$

or, as with (11),

$$\sum_{t=1}^{t=T} MU_{inc}^t [(MRS)_t - p_t] = 0. \tag{21}$$

Equation (21) implies that the consumer optimizes with respect to each period's price ratio (recall that the numeraire price for each period is set equal to 1), as well as with respect to the overall debt level $\sum_{t=1}^{t=T} z_t (1 + r)^{1-t}$.

There are several implications for purchasers of housing services. First, the ability to access capital markets by borrowing and/or lending may increase consumers' utilities and change housing purchases. Ability to borrow may change not only the timing but also the amount of the purchases.

This same logic applies to a comparison of optimal VPMs to fixed-payment mortgages (FPMs). In a well-functioning capital market, borrowers could draw upon future income as necessary to adjust housing purchases and expenditures. Since most borrowers are unable to borrow perfectly against future income, VPMs allow them to adjust more precisely at the margin of the housing and nonhousing prices that must be paid. As such, utilities rise because the constraint that $z_1 = z_2 = \cdots = z_T$ is lifted. The order of magnitude of the surplus to borrowers and lenders depends on the nature of the borrowers' utility functions, prices, and incomes.

## 5. AN ANALYTICAL SOLUTION FOR A TWO-PERIOD MODEL

Although an equilibrium solution does exist, and is unique, for a multiperiod problem, analytical solutions are not readily tractable for multiperiod models. In this section, we use a two-period model to analyze the impacts of changes in incomes, prices, and/or preferences. Our framework explicitly models the endogeneity of the moving decision. Consider the decision whether to move or not, given the transactions costs of moving

(1) when optimizing each period, but having to pay moving costs,

or

(2) when optimizing over two periods, without moving costs.

This section develops an expression for dollar value $\nu$ of utility lost by staying in the same location under changed economic conditions. Given constant moving cost $m$, an increase in $\nu$ implies an increased likelihood of moving.

Consider the separable utility function where parameter $\theta$ reflects heterogeneity of tastes. If $\theta = 1$, tastes are the same in each period. If $\theta$ is greater (less) than 1, housing is preferred more (less) in the future relative to the composite consumption good:[8]

$$U^* = \gamma \ln c_1 + \gamma/(1+\delta) \ln c_2 + \beta \ln h_1 + \theta\beta/(1+\delta) \ln h_2. \quad (22)$$

Assume, further, perfect capital markets such that

$$\begin{aligned} y^* &= y_1 + y_2/(1+r) - \nu/(1+r) \\ &= c_1 + p_1 h_1 + c_2/(1+r) + p_2 h_2/(1+r). \end{aligned} \quad (23)$$

Solving for $c_t$, $h_t$, we obtain

$$c_1^* = y^*/E, \quad (24)$$

$$c_2^* = [(1+r)/(1+\delta)](y^*/E), \quad (25)$$

$$h_1^* = (\beta/\gamma p_1)(y^*/E), \quad (26)$$

$$h_2^* = (\theta\beta/\gamma p_2)[(1+r)/(1+\delta)](y^*/E), \quad (27)$$

[8] Here, $\gamma + \beta < 1$; $\gamma + \theta\beta < 1$.



where

$$E = [1 + 1/(1+\delta)] + (\beta/\gamma)[1 + \theta/(1+\delta)]. \quad (28)$$

Alternatively, if households do not move, $h_1 = h_2$, or

$$U^{**} = \gamma \ln c_1 + \gamma/(1+\delta) \ln c_2 + \beta[1 + \theta/(1+\delta)] \ln h, \quad (29)$$

subject to

$$y^{**} = y_1 + y_2/(1+r) = c_1 + c_2/(1+r) + [p_1 + p_2/(1+r)]h. \quad (30)$$

Solving for $c_1$, $c_2$, $h$, yields

$$c_1^{**} = y^{**}/E, \quad (31)$$

$$c_2^{**} = [(1+r)/(1+\delta)](y^{**}/E), \quad (32)$$

$$h^{**} = (\beta/\gamma)(1 + \theta/(1+\delta))/[p_1 + p_2/(1+r)](y^{**}/E). \quad (33)$$

To determine the cost of the moving constraint, let $K = \gamma E$ and solve for $\nu$ such that $U^* = U^{**}$.[9]

The equilibrium value of $\nu$ is

$$\nu = -(1+r)(Q_1 + Q_2)y^{**}/[K(1 - Q_1 - Q_2)], \quad (34)$$

where

$$Q_1 = \beta \ln\{[1 + \theta/(1+\delta)]p_1/p^*\},$$

and

$$Q_2 = \beta\theta/(1+\delta) \ln\{[(1+\delta)/(1+r)][1 + \theta/(1+\delta)][p_2/(\theta p^*)]\}.$$

If $\nu > 0$, then $\partial\nu/\partial y^{**} > 0$. It is also easily shown that

$$\begin{aligned} &\partial\nu/\partial p_1 < 0,\ \partial\nu/\partial p_2 > 0,\ \partial\nu/\partial r < 0,\ \partial\nu/\partial\delta > 0; \\ &\qquad \text{as } p_2/p_1 > \theta(1+r)/(1+\delta). \end{aligned} \quad (35)$$

Evaluating the effect of changed heterogeneity parameter $\theta$, we obtain

$$\partial\nu/\partial\theta > 0; \qquad \text{as } \theta > [(1+\delta)/(1+r)](p_2/p_1). \quad (36)$$

[9] Goodman (1992) derives the comparative statics results in more detail.

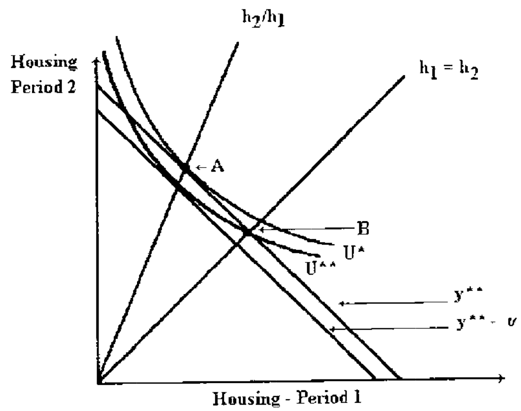


FIG. 1. The impact of the immobility constraint.

These results can also be demonstrated in Fig. 1. Consider the choice of housing amounts $h_1$ and $h_2$ with no moving costs; i.e., point A. From Eqs. (26) and (27), the ratio $h_2/h_1$ is on a ray with slope $[(1 + r)\theta p_1]/[(1 + \delta)p_2]$. The constraint that $h_1 = h_2$ is represented by the 45° line from the origin. The moving cost equivalent is that $\nu$ necessary to make the unconstrained indifference curve pass through point B, which is where the 45° line cuts the original income constraint $y^{**}$. The more binding the constraint, i.e., the more the original ray differs from 45°, the greater the loss of utility, the higher the value of $\nu$, and the more probable will be a move between Periods 1 and 2.

The implications for multiple periods follow directly. Changes in prices, incomes, and preferences may affect demand and/or mobility in stays prior or subsequent to the actual change. Moving one period earlier or later changes the vector of incomes and prices, as well as the degree of taste heterogeneity in prior or subsequent stays. Increased income in a given period, when capital markets are not perfect, may essentially change the degree of homogeneity of preferences within a given stay, depending on when within the stay it occurs.

## 6. LENDERS' DECISIONS

In our model lenders provide resources in the following manner: Both the borrower and the lender know what the vector of future housing and nonhousing prices will be. The lender has determined that the borrower's



future income is sufficient to service the debt for a term of $\tau$ periods, after which the borrower will move to another home.[10] To live in the desired house $h_0$, a borrower desires a loan that covers the present discounted value of future payments. The lender is willing to make the loan as long as the present discounted value of the future payments equals loan value $L$ such that

$$L = h_0 \sum_{t=1}^{t=\tau} p_t(1 + r)^{1-t} = z \sum_{t=1}^{t=\tau} (1 + r)^{1-t} = zF. \tag{37}$$

If a risk-neutral lender is willing to make a fixed-payment loan $L$ for term $\tau$, she should be willing to accept any guaranteed stream of variable payments $V$ such that

$$V = h_0 \sum_{t=1}^{t=\tau} p_t(1 + r)^{1-t} = \sum_{t=1}^{t=\tau} z_t(1 + r)^{1-t} = L. \tag{38}$$

As noted above, with a VPM the optimal loan may change because of the consumer's opportunity to optimize.[11]

The increased consumer utility due to the ability to make variable payments represents a surplus, the distribution of which could be negotiated between lenders and borrowers. That is, one can calculate the equivalent net present value to the consumer of the ability to pay at variable rates. It might be argued that the *fixed* payments serve as signals to the lenders that the borrowers are creditworthy and can be counted on to pay on time. Yet certainly some portion of the surplus could serve to insure lenders against the additional risk, even in the case of risk-averse lenders, and still allow the borrowers to increase their utility. The order of magnitude of such a surplus is important, and we turn to simulations to address this issue.

## 7. SIMULATION

The simulation uses a 15-period model in which each period signifies 1 year. Both housing and the composite consumption good are purchased as streams of services. Although the model permits utility functions and

[10] At this point, we assume that uncertainty and asymmetric information are not at issue. We address both below.

[11] As with the public good equilibrium, housing demand may increase or decrease due to the variable payment mortgage, depending on the consumer's preferences, the interest rate, and the discount rate.

discount rates to vary over time, for our simulations they are held constant. We calculate the consumer optimum for a fixed payment mortgage (FPM) and compare it to the optimum for the optimal variable payment mortgage (VPM).

For the simulations presented, we use a CES utility function of the form

$$U_t = A_t[\beta_t h^{\rho_t} + \gamma_t c_t^{\rho_t}]^{\xi_t/\rho_t}. \quad (39)$$

The elasticity of substitution $\sigma = 1/(1 - \rho)$. For the simulations, $\sigma$ is set at 0.5 ($\rho = -1.0$).[12] The term $\xi_t$ reflects diminishing marginal utility, and is set at 0.9; $\beta_t$ is always normalized such that housing share of income is 30% at $p_t = 1$. Utility at time $t$ is discounted by the discount rate $D_t^{1-t}$.

We chose to represent the effect that the VPM would have had on housing consumption patterns in the United States. Incomes and prices were gathered for a 15-year period between 1974 and 1988. The income measure used was the median income of a U.S. household whose head was aged 15 to 24 in 1974 and 35 to 44 in 1988. In 1988 dollars, these respective values were $24,700, and $36,550 (see Henson, 1990). Intermediate values were linearly imputed.

Housing price was specified from the Consumer Price Index, and the Shelter Price Index, assuming that housing was a 30% share of all expenditures. This provides a parameter for housing prices relative to the prices of all other goods, as implied in the optimization.[13] Moving transactions costs were arbitrarily set at 1.0. With the starting income of 24.70, this implies costs that are about 4% of a single year's income.

Tables I through III consider the cases with interest rate $r$ equal to ($r = 0.02$), less than ($r = 0.01$), or greater than ($r = 0.03$) the discount rate ($\delta = D - 1$) of 0.02. With $\delta > 0$, a FPM makes the consumer pay more for housing at the beginning of the mortgage than she would wish. The gain from the VPM over the FPM, with $\delta > 0$, is the ability to push purchasing power forward; the cost is the interest rate on the borrowed resource. Hence, the lower interest rate $r$ relative to discount rate $\delta$, the larger the potential gain from a VPM.

Begin first in Table I with $\delta = r$. In a FPM simulation the consumer's utility maximization would include a move at the beginning of Year 9 (as well as the initial move in Year 1), increasing housing consumption by 14.0% (9.697/8.506), and housing payment by 26.8% (10.127/7.984). The fixed payment mortgage yields a utility level of 103.7067. Even though the consumer incurs transactions costs by moving in Year 9, the utility level achieved is higher than the level of 103.6077 (noted at the bottom of Table I), which would have accrued with no additional move. Moving, even with transactions costs, serves as a substitute for the capability of fully accessing capital markets.

The availability of a VPM for a single stay indicates that the individual would pay considerably less for housing in the first part of the stay. The Year 1 VPM payment of 2.714 in Table I is 66% less than the Year 1 FPM payment, and the Year 8 payment is 16% higher. The consumer with a

[12] Increases in $\sigma$ reduce mobility because they make the nonhousing good $c$ a better substitute for $h$. Hence, constraints on $h$ are less binding. Numerous simulations have been run for various values of $\sigma$ and for various values of moving costs. Recall also that the general results do not depend on C.E.S. functions, and that any twice differentiable utility function could be used.

[13] As noted by Polinsky (1977), this is also the appropriate means for estimating housing demand.



TABLE I
Optimization with Interest Rate = Discount Rate

Parameters: Discount = 1.02
Interest = 1.02
FPM: Utility = 103.7067 (move in Year 9)
VPM: Utility = 104.2607
Housing: Hous = 9.066

$S_f = 2.060$ (Equivalent variation)

$S_v = 2.310$ (Compensating variation)

| T | INC | HP | Fixed payment mortgage CON | Fixed payment mortgage HOUS | Fixed payment mortgage PAY$^F$ | Variable payment mortgage PAY$^V$ | Variable payment mortgage CON$^V$ |
|---|---|---|---|---|---|---|---|
| 1 | 24.70 | 1.00 | 15.713 | 8.506 | 7.984 | 2.714 | 20.984 |
| 2 | 25.49 | 0.87 | 17.503 | 8.506 | 7.984 | 4.504 | 20.984 |
| 3 | 26.28 | 0.87 | 18.293 | 8.506 | 7.984 | 5.294 | 20.984 |
| 4 | 27.07 | 0.87 | 19.083 | 8.506 | 7.984 | 6.083 | 20.984 |
| 5 | 27.86 | 0.90 | 19.873 | 8.506 | 7.984 | 6.873 | 20.984 |
| 6 | 28.65 | 0.93 | 20.663 | 8.506 | 7.984 | 7.663 | 20.984 |
| 7 | 29.44 | 1.08 | 21.453 | 8.506 | 7.984 | 8.453 | 20.984 |
| 8 | 30.23 | 0.99 | 22.243 | 8.506 | 7.984 | 9.243 | 20.984 |
| 9 | 31.02 | 1.01 | 19.890 | 9.697 | 10.127 | 10.033 | 20.984 |
| 10 | 31.81 | 0.99 | 21.680 | 9.697 | 10.127 | 10.823 | 20.984 |
| 11 | 32.60 | 1.00 | 22.470 | 9.697 | 10.127 | 11.612 | 20.984 |
| 12 | 33.39 | 1.03 | 23.260 | 9.697 | 10.127 | 12.402 | 20.984 |
| 13 | 34.18 | 1.08 | 24.050 | 9.697 | 10.127 | 13.192 | 20.984 |
| 14 | 34.97 | 1.10 | 24.840 | 9.697 | 10.127 | 13.982 | 20.984 |
| 15 | 36.55 | 1.11 | 26.427 | 9.697 | 10.127 | 15.569 | 20.984 |

*Note.* Optimum under 15-year FPM: $U = 103.6077$; Hous = 9.034; Payments = 8.890.

TABLE II
Optimization with Discount Rate Greater Than Interest Rate

| | | | |
|---|---|---|---|
| Parameters: | Discount | = | 1.02 |
| | Interest | = | 1.01 |
| FPM: | Utility | = | 103.6850 (move in Year 9) |
| VPM: | Utility | = | 104.7538 |
| Housing: | Hous | = | 9.109 |

$S_f$ = 4.100 (Equivalent variation)

$S_v$ = 4.770 (Compensating variation)

| | | | Fixed payment mortgage | | | Variable payment mortgage | |
|---|---|---|---|---|---|---|---|
| T | INC | HP | CON | HOUS | $PAY^F$ | $PAY^V$ | $CON^V$ |
| 1 | 24.70 | 1.00 | 15.711 | 8.502 | 7.986 | 0.480 | 23.217 |
| 2 | 25.49 | 0.87 | 17.501 | 8.502 | 7.986 | 2.593 | 22.894 |
| 3 | 26.28 | 0.87 | 18.291 | 8.502 | 7.986 | 3.704 | 22.573 |
| 4 | 27.07 | 0.87 | 19.081 | 8.502 | 7.986 | 4.814 | 22.254 |
| 5 | 27.86 | 0.90 | 19.871 | 8.502 | 7.986 | 5.921 | 21.936 |
| 6 | 28.65 | 0.93 | 20.661 | 8.502 | 7.986 | 7.026 | 21.621 |
| 7 | 29.44 | 1.08 | 21.451 | 8.502 | 7.986 | 8.130 | 21.307 |
| 8 | 30.23 | 0.99 | 22.241 | 8.502 | 7.986 | 9.232 | 20.995 |
| 9 | 31.02 | 1.01 | 19.887 | 9.692 | 10.130 | 10.332 | 20.685 |
| 10 | 31.81 | 0.99 | 21.677 | 9.692 | 10.130 | 11.430 | 20.377 |
| 11 | 32.60 | 1.00 | 22.467 | 9.692 | 10.130 | 12.526 | 20.071 |
| 12 | 33.39 | 1.03 | 23.257 | 9.692 | 10.130 | 13.621 | 19.766 |
| 13 | 34.18 | 1.08 | 24.047 | 9.692 | 10.130 | 14.713 | 19.464 |
| 14 | 34.97 | 1.10 | 24.837 | 9.692 | 10.130 | 15.804 | 19.163 |
| 15 | 36.55 | 1.11 | 26.424 | 9.692 | 10.130 | 17.690 | 18.864 |

*Note.* Optimum under 15-year FPM: $U$ = 103.5309; Hous = 9.018; Payments = 8.890.

FPM would then increase housing demand and payments in Year 9, but the VPM would still be higher (in all but Year 9).

In measuring the potential gains from the VPM, note the "index number problem" in measuring the wealth variation that equates the utility level. One can either calculate the *equivalent variation* $S_f$ necessary with the FPM to bring the homebuyer *up* to the VPM utility, or the *compensating variation* $S_v$ necessary in the VPM to bring the homebuyer *down* to the FPM utility.[14]

[14] See Varian (1984, pp. 264–265) for discussion of these two concepts. We adopt the convention of referring to both in positive terms. Schwab (1982) uses compensating variation.



TABLE III
Optimization with Discount Rate Less Than Interest Rate

| | | | |
|---|---|---|---|
| Parameters: | Discount | = | 1.02 |
| | Interest | = | 1.03 |
| FPM: | Utility | = | 103.7279 (move in Year 9) |
| VPM: | Utility | = | 103.9478 |
| Housing: | Hous | = | 9.0483 |

$S_f$ = 0.780 (Equivalent variation)

$S_v$ = 0.930 (Compensating variation)

| | | | Fixed payment mortgage | | | Variable payment mortgage | |
|---|---|---|---|---|---|---|---|
| T | INC | HP | CON | HOUS | $PAY^F$ | $PAY^V$ | $CON^V$ |
| 1 | 24.70 | 1.00 | 15.716 | 8.510 | 7.982 | 4.776 | 18.921 |
| 2 | 25.49 | 0.87 | 17.506 | 8.510 | 7.982 | 6.271 | 19.216 |
| 3 | 26.28 | 0.87 | 18.296 | 8.510 | 7.982 | 6.764 | 19.513 |
| 4 | 27.07 | 0.87 | 19.086 | 8.510 | 7.982 | 7.255 | 19.812 |
| 5 | 27.86 | 0.90 | 19.876 | 8.510 | 7.982 | 7.744 | 20.113 |
| 6 | 28.65 | 0.93 | 20.666 | 8.510 | 7.982 | 8.232 | 20.415 |
| 7 | 29.44 | 1.08 | 21.456 | 8.510 | 7.982 | 8.717 | 20.720 |
| 8 | 30.23 | 0.99 | 22.246 | 8.510 | 7.982 | 9.201 | 21.026 |
| 9 | 31.02 | 1.01 | 19.893 | 9.702 | 10.124 | 9.683 | 21.334 |
| 10 | 31.81 | 0.99 | 21.683 | 9.702 | 10.124 | 10.163 | 21.644 |
| 11 | 32.60 | 1.00 | 22.473 | 9.702 | 10.124 | 10.642 | 21.955 |
| 12 | 33.39 | 1.03 | 23.263 | 9.702 | 10.124 | 11.118 | 22.269 |
| 13 | 34.18 | 1.08 | 24.053 | 9.702 | 10.124 | 11.593 | 22.584 |
| 14 | 34.97 | 1.10 | 24.843 | 9.702 | 10.124 | 12.066 | 22.901 |
| 15 | 36.55 | 1.11 | 26.430 | 9.702 | 10.124 | 13.334 | 23.220 |

*Note.* Optimum under 15-year FPM: $U$ = 103.6830; Hous = 9.050; Payments = 8.882.

$S_v$ will generally be larger (in absolute value) than $S_f$. Consider a dollar supplement in the fixed payment regime. It is applied to a FPM and supplements a weighted utility measure. Consider then the dollar deduction from the VPM. The consumer will be able to reduce payment in order to minimize the loss of utility. Since a dollar loss with the VPM sacrifices less utility at the margin than a dollar increase gains, it takes a larger absolute fall in income under a VPM than absolute rise under a FPM. This suggests that $S_f$ and $S_v$ supply lower and upper bounds to the welfare gain from the VPM.

At $\delta = r = 0.02$, $S_f = 2.060$ and $S_v = 2.310$. It is wise to put this into perspective. These amounts are more than double the moving costs (1.0) and equivalent to 8.3% (for $S_f$) or 9.4% (for $S_v$) of the first year's income.

Alternatively, they are 25.8% (for $S_f$) to 28.9% (for $S_v$) of the initial housing payment under a fixed regime.

Recall that the lender's decision relates the borrower's VPM to the decision to lend with a FPM. $S_f$ and/or $S_v$ reflect a potential surplus to a lender. Reiterating concern about adverse selection, there is a "cushion" for a lender to use for "insurance" against the probability of adverse selection.

Tables II and III consider the results with interest rates of 0.01 and 0.03, respectively. As expected, holding $\delta$ constant, the benefits of a VPM decrease with $r$. With $r = 0.01$, the value of $S_f(S_v)$ is 4.100 (4.770), or 16.6 and 19.3%, respectively, of first year's income. With $r = 0.03$, the value of $S_f(S_v)$ is 0.780 (0.930). Clearly, the higher the value of $r$ relative to the discount rate, the less desirable the VPM. Moreover, the potential surplus available to the lender for insurance against adverse selection grows as the individual's discount rate increases relative to the market interest rate.

The ability to shift forward purchasing power also increases housing demand. In the presence of heterogeneous preferences and income changes, the ability to vary mortgage payments has larger payoffs and will increase housing demand. This follows from the likelihood that constraint that $z_1 = z_2 = \cdots = z_t = z$ in the fixed mortgage would become increasingly costly. For example, in Table I with the $\delta = r$, the ability to vary payments increases the housing demand by about 6.6% (9.066/8.506) over the initial stay with the FPM. The increment is even higher when $\delta > r$ (7.1% in Table II).

## 8. CONCLUSIONS

This study has considered the development of optimal mortgages from the multiperiod optimization on the part of both borrowers and lenders. The borrower's decision has many elements of a public good allocation. In allocating a public good, the main problem is getting consumers to reveal their "true" preferences. In the choice of a VPM schedule, the consumer can *improve* her well-being by matching housing payments to her preferences. A VPM offers two ways for the borrower to increase utility by redistributing purchasing power from period to period:

1. to link marginal utility of income over time through the interest and the discount rates.
2. to address the varying prices of housing and the numeraire good.

The absence of perfect capital markets makes the redistribution of income from period to period without a VPM difficult. The generally lower first-period payment under a VPM (which occurs upon the purchase of the stream of housing services) can be treated as a housing down payment. As such, the model makes the down payment endogenous. Considering the first VPM payment in this manner, we show that lenders can benefit from lowered down payments, even without the increased interest rates that are assumed in other models.



Most importantly, the model provides useful insights into why VPMs could be beneficial to both lenders and borrowers. It suggests that the development of an optimal payment schedule could lead to increased profits for the former, and increased utility for the latter.

We have attempted to be relatively conservative in our choice of simulation parameters, and we have still derived substantial surpluses. As our results indicate, the surplus increases as the individual discount rate rises relative to the interest rate. This suggests that although a VPM will not be appropriate for everyone, it will be highly valued by prospective homeowners who discount the future relative to the market.

Mortgage lenders may cite the problem of asymmetric information between lenders and borrowers as the main hindrance to the implementation of our VPM instrument. We respond in two ways. First, the lender who would approve a fixed-payment instrument would *already* have addressed many of the information concerns in approving the loan. Second, we have demonstrated that the potential surpluses are large enough to justify an "up-front" percentage payment (points) to insure against nonpayment, or a system of late-payment penalties. Both would encourage borrowers to present their financial situations accurately to lenders. In cases such as those analyzed, then, VPM mortgages must be considered Pareto superior to the FPM mortgages which have been so popular in the post-World War II era.

Further research would address the issues of uncertainty and the roles of nonhousing decisions such as the household's portfolio choice decision. In our model, both households and lenders have perfect knowledge regarding all future parameters and act accordingly. Uncertainty would affect borrowers' precautionary savings and the use of prepayment options. It would affect lenders' concerns about mortgage default with respect to possible adverse selection in the issuing of loans. With portfolio choice theory, the consumer would choose the amount and type of mortgage along with the rest of her portfolio. Both extensions would broaden the analyses yielded by the model proposed here.

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