

## DEMOGRAPHICS OF INDIVIDUAL HOUSING DEMAND

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There has been little systematic treatment of demographic variables in the housing literature. This paper reviews methods used to include demographic variables in demand systems. It then discusses their inclusion in a system of housing demand equations in which tenure choice and demand are jointly determined and estimated. Explanatory power is enhanced substantially in both the tenure choice and the demand regressions by demographic variables. Elasticities at means are not substantively changed by demographic variables, but elasticities *away from the means* are sensitive to their inclusion. Blacks are slightly *more* likely to own than whites, all else equal; white demand, however, is 4.5 to 26.5 percent higher than black demand.

### 1. Introduction

There has been little systematic treatment of demographic variables in the estimation of housing demand. Although most analysts would agree that big households may purchase more housing than smaller ones, blacks may purchase less housing than whites due to discrimination, and price and income elasticities may vary according to age, for example, the examination of demographic effects has not been rigorous.

This study addresses estimates of owner and renter demand jointly with tenure choice, with special attention to demographic variables. Goodman (1988) shows how market housing demand can be expressed as a function of the probability of owning vs renting, and the differential quantities of owner and renter housing purchased. This study explicitly models the demographic interactions occurring both in the tenure choice and in the resulting demand estimates.

The paper begins with a brief review of the methods used to include demographic variables in demand systems. It then discusses the inclusion of demographic variables in a system of housing demand equations in which tenure choice and demand are jointly determined. Empirical results are then presented. There are three major findings in examining the empirical results.

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First, in both the probit (tenure choice) and the subsequent demand regressions, explanatory power is enhanced substantially. In the tenure choice, moving from simpler to more complex formulations leads statistically to more significant relationships; moreover, the number of 'wrong' predictions decreases by 20%.

Second, in the demand estimates, although elasticities at the means are not substantively different when demographic variables are included, the demographic interactions change elasticities substantially *away* from the means. Since many of the social policy implications of demand estimates relate to populations that are situated away from the general population means (incomes, for example, are most often substantially below the means) careful estimation is necessary in order to avoid faulty conclusions.

Third, the paper analyzes how tenure choice adjustments may change elasticity measurements. Olsen (1987), in a life-cycle analysis, shows how income elasticities, given housing tenure, are likely to rise with age. I find, however, that as age increases, the probability of owning (and hence of consuming larger amounts of housing) increases. This increases the base on which elasticity is calculated, thus offsetting the tenure-specific increases in income elasticity, and leading to a *decrease* in the total income elasticity of housing demand.

## 2. Demographic impacts on demand

There are two aspects of the impacts of demographic variables on housing demand. The first is subsumed under the general descriptor of *tastes*. Large families (presumably with more children) may have tastes for more housing. Blacks, for example, may have tastes for less housing, or may be excluded due to discriminatory policies from large and/or expensive housing. Failure to account for these differences implies that all households, irrespective of demographics, have the same utility (and hence, demand) functions. Such may be the case, but it is a very restrictive maintained assumption.

The second impact involves the treatment of demographic variables in the optimization process. The housing purchase at a given time is related both to current and to future prices and incomes. This is due both to the intertemporal life-time optimization that characterizes the purchase of many types of goods, *and* to the high transactions costs of changing the amount of housing that is purchased.<sup>1</sup> Olsen (1987) provides a simple but useful model, in which a household optimizes over a finite horizon, based on a set of known future incomes and future prices. In this framework it is easily seen that elasticities with respect to current income and current price depend on

<sup>1</sup>Even if the underlying utility function is separable over time, the existence of positive moving costs makes it necessary to include future incomes and prices in the optimization process.

the time remaining to the horizon. In an analysis similar to the life-cycle model, the impacts (and hence elasticities) of the price and/or income changes increase with age (and hence, decreasing time to the horizon).

Several forms for the inclusion of demographic variables have been proposed. Consider a simple linear demand equation, involving only the vectors of relevant prices and incomes. *Translation* involves the addition of terms that do not interact with the price and income terms of the skeleton regression. Thus parameter estimates are invariant to the demographic characteristics, but elasticities, of course, can vary.

*Scaling* redefines the parameters of the demand system as functions of household demographic variables. The two can be combined by first scaling and then translating (often referred to as a *Gorman* specification), or by translating, then scaling [defined by Pollak and Wales (1981) as a *reverse Gorman* specification]. In the linear demand regression, these forms are nested, and permit rigorous statistical testing.

Pollak and Wales note that utility function parameters can be allowed to vary with demographic characteristics. The present analysis does not use this approach for two reasons. First, it is unclear how the approach performs in a system where tenure and demand are jointly determined and the decision to buy has investment aspects.<sup>2</sup> Second, even within the literature that examines housing alone, attempts to implement utility maximization models (typically Stone-Geary functions) have been only modestly successful.<sup>3</sup>

Consider, then, the formulation of a demand and/or tenure choice (owner or renter) indicator  $I$ :

$$I = \alpha Y + \beta P + \theta_1 D + \theta_2 DY + \theta_3 DP + \varepsilon, \quad (1)$$

where  $Y$  is the appropriate income (permanent/transitory or current) vector,  $P$  is the relevant price vector (including prices of alternatives, and/or user cost components), and  $D$  is the vector of demographic terms. In the simple system,  $\theta_1 = \theta_2 = \theta_3 = 0$ . Linear translation restricts parameters  $\theta_2$  and  $\theta_3$  to 0. Linear scaling restricts  $\theta_1$  to 0, while Gorman's method (which involves scaling, then translating) allows all three parameters to vary. The various formulations in (1) are nested within the Gorman (henceforth, *extended*) formulation, allowing testing for the inclusion of variables (in the probit,

<sup>2</sup>King (1981), and others, have modeled housing consumption only, using a utility maximization approach.

<sup>3</sup>Often, for example, the so-called 'subsistence parameters' in these models are estimated to be negative. See Reeder (1985), or Friedman and Weinberg (1982). There has been no systematic examination of the role of demographic indicators. For comments and further discussion, see Mayo (1981).

using likelihood ratio tests, and in the linear models, using standard  $F$ -tests).<sup>4</sup>

Eq. (1) provides a useful format for classifying the treatment of demographic effects in other housing demand studies. A selected set is summarized in table 1.<sup>5</sup> Demographic variables usually include elements of a set including age, race, sex, education, and marital status of the household head, household size, and number of children. Age is often included quadratically [e.g. Ihlandfeldt (1981)], and demographic variables can be interacted [Kain and Quigley (1972)]. Ihlandfeldt also considers the *expectation* of new children. Most of the studies use translation methods ( $\theta_2 = \theta_3 = 0$ ), although two use scaling ( $\theta_1 = 0$ ), and Dynarski and Sheffrin (1985) use Gorman-type methods. Age, if significant, is usually positively related to quantity, race (i.e. black) is usually negatively related. None of the studies systematically examines and compares the various demographic formulations sketched out above.

### 3. A general formulation

Goodman (1988) develops a general framework for demand estimation which includes both tenure choice and demand equations:

$$H(Q) = fQ_o + (1-f)Q_r \quad (2)$$

$f$  [or  $(1-f)$ ] can be interpreted as the probability of owning [renting] (or percentage owning [renting]), and  $Q_o$  [ $Q_r$ ] is the amount of owner [renter] housing demanded.  $Q_o$ ,  $Q_r$ , and  $f$  are all functions of the price, income, and demographic variables. Full differentiation of (2) gives elasticities with respect to income, prices and demographic variables.

The tenure choice equation is related to income and demographic variables, to the relative price ratio of owner ( $P_o$ ) and renter ( $P_r$ ) housing (to assure homogeneity) and to the value-rent ratio  $V$  (as a measure of asset viability).<sup>6</sup> Thus,

<sup>4</sup>The scaling model and the Gorman model both provide higher order quadratic terms involving the demographic variables. They are omitted here, and in the empirical work for several reasons. First, as noted by Pollak and Wales, (1981) the scaling and translation parameters in this model cannot be identified, and the additional terms do not help. Second, experiments with these terms, and indeed even with some of the prescribed linear interactions, lead to multicollinearity problems in estimation. With dummy demographic variables, such as race or sex, the matrix becomes singular. One might further argue that stratification by the appropriate demographic categories may be appropriate. In all but the most massive data sets, such an approach may lead to extremely small sample sizes for individual cells.

<sup>5</sup>Mayo (1981) surveys some of the earlier work.

<sup>6</sup>The value-rent ratio relates asset value to the rent that would have been charged, with a high value-rent ratio suggesting a market-indicated expectation of a capital gain. It is thus a component of the user cost of housing. Goodman (1988) provides a more complete presentation of the derivation and analysis of the model used here.

Table 1  
Selected articles using demographic methods.

Author(s)	Date	Type	Age	Race <sup>a</sup>	Sex <sup>b</sup>	Mar <sup>c</sup>	Hhsize	Educ.	Other	Method
Barnett/Noland	1981	Owners	+ <sup>d</sup>	+	NA	+	+	0	# child	t <sup>e</sup>
		Renters	+	0	NA	0	+	0	# child	t
Börsch-Supan/ Pollakowski	1986	Owners	+	NA	NA	+	+	NA		t
		Renters	+	NA	NA	0	+	NA		t
Carliner	1973	Owners	+/-	-	-	NA	NA	NA		t
Dynarski/ Scheffrin	1985	Ten-Fst <sup>f</sup>	0	0	+	NA	-	NA	# child	t
		Ten-Ex	-	0	0	NA	0	NA	# child	t, st
Follain	1982	Tenure	+	NA	NA	NA	+	0	age <sup>2</sup>	t
		All	-	NA	NA	NA	+	0	age <sup>2</sup>	t
Friedman/ Weinberg	1982	All	NA	-	NA	0	0	NA		s
Goodman/ Kawai	1982	Owners	NA	-	NA	NA	-	NA	# child	t
Henderson/ Ioannides	1987	Tenure	+	-	0	+	+	0		t
		Owners	0	0	NA	NA	0	0		t
		Renters	0	0	NA	NA	0	+		t
Ihlandfeldt	1981	Own-move	0	-	+	NA	0	0	exp chd	t
		Rent-move	+	0	+	NA	+	+	exp chd	t
		Own- nonmove	+	-	0	NA	0	+	exp chd	t
		Rent- nonmove	+	-	+	NA	+	-	exp chd	t
Ihlandfeldt	1982	Renters	NA	-	NA	NA	+	NA		s
Kain/Quigley	1972	Tenure	+	-	+	+	-	0	Inter <sup>g</sup>	t
Rosen	1979	Tenure	+	-	-	NA	+	NA		t
		All	+	0	+	NA	0	NA		t

<sup>a</sup>1 if black.

<sup>b</sup>1 if male.

<sup>c</sup>1 if married.

<sup>d</sup>+, - = significantly positive (negative); 0 = insignificant; NA = not applicable.

<sup>e</sup>t = translation; s = scaling.

<sup>f</sup>Tenure choice: First-time owners; Existing owners.

<sup>g</sup>Interaction of demographic variables.

$$f = f(Y, P_o/P_r, V, D). \quad (3)$$

Owner demand is a function of income, owner-price (relative to all other goods), value-rent ratio (the two serving together as user cost), and demographic variables:

$$Q_o = Q_o(Y, P_o, V, D). \quad (4)$$

The 'user cost' interpretation of housing demand suggests that the demand

for rental services should not include the value-rent term, as it reflects change in asset value rather than service flow, implying:

$$Q_r = Q_r(Y, P_r, D). \quad (5)$$

Specification of the right-hand side of (2) with (3)–(5) yields the following elasticities:

$$\eta_Y^* = f q_o \eta_{Q_o, Y} + (1-f) q_r \eta_{Q_o, Y} + (q_o - q_r) \eta_{f, Y}, \quad (6a)$$

$$\eta_{P_o}^* = f q_o \eta_{Q_o, P} + (q_o - q_r) \eta_{f, P}, \quad (6b)$$

$$\eta_{P_r}^* = (1-f) q_r \eta_{Q_r, P} - (q_o - q_r) \eta_{f, P}, \quad (6c)$$

$$\eta_V^* = f q_o \eta_{Q_o, V} + (q_o - q_r) \eta_{f, V}, \quad (6d)$$

$$\eta_D^* = f q_o \eta_{Q_o, D} + (1-f) q_r \eta_{Q_r, D} + (q_o - q_r) \eta_{f, D}, \quad (6e)$$

where  $q_o = Q_o/H$ , and  $q_r = Q_r/H$ . The right-hand side elasticities of (6a)–(6e) are partial elasticities, *conditional* on tenure choice.

A brief discussion of these terms is useful. Single equation estimates of owner (renter) demand restrict  $f$  to 1 (0), and the differential impacts of the tenure choice decisions rest on the premise that  $Q_o$  and  $Q_r$  differ. The tenure choice effects of equal percentage changes in  $P_o$  and  $P_r$  negate each other, as noted by adding (6b) and (6c). The impacts of terms  $D$  occur both in the demand and tenure choice equations and in their impacts on other variables. Elasticities  $\eta_{Q_o, D}$ ,  $\eta_{Q_r, D}$ , and  $\eta_{f, D}$  may also include the impacts of variables  $D$  on 'economic variables' such as income. For example,

$$\eta_{Q_o, D} = (\eta_{Q_o, D} + \eta_{Q_o, Y} \eta_{Y, D}) \quad (6f)$$

if  $Y$ , representing permanent income, is a function of age, a demographic variable.

This system of equations is logically separate from any explicit econometric considerations. Still, most analysts [see, for example, Lee and Trost (1978) and Rosen (1979)] agree that the error terms in demand eqs. (4) and (5) might be related to the error term in tenure choice (3), and 'Heckit' forms of estimation [Heckman (1979)] have become common. As a result, estimates of  $\lambda_o = \phi/\Phi$ , and  $\lambda_r = -\phi/(1-\Phi)$ , where  $\phi$  represents the probability density function and  $\Phi$  represents the cumulative density function underlying probit regression (3), are inserted in (4) and (5), respectively.

Consider the joint maximization of the three likelihood functions:

$$L^* = L_3(D)L_4(D)L_5(D), \quad (7a)$$

or in logs:

$$L^{**} = \ln L_3(D) + \ln L_4(D) + \ln L_5(D), \quad (7b)$$

where the subscripts refer to the underlying eqs. (3) through (5). Lee and Trost show how the joint-likelihood function (7b) may be estimated *directly* with full information maximum likelihood methods, rather than the two-stage procedures suggested by Heckman. In my study, however, repeated attempts to maximize across *all* three equations were unsuccessful at maximizing the relevant function, and further examination of the problem suggested that the convergence problem lay in the *renter* equation (which, otherwise, seemed well-behaved). In the empirical results presented below, the tenure choice (3) and owner demand (4) equations are estimated using maximum likelihood methods, and the renter demand equation (5) is estimated *conditional* on the maximum likelihood probit and owner demand estimates from (3) and (4).<sup>7</sup>

#### 4. Estimates

##### 4.1. Probit equations

This section applies the methods from (1) to probit equation (3). There are several reasons to suspect that demographic variables may affect tenure choice. Age of household head (aside from its independent impact on permanent income) may reflect tastes for housing, as may household size. Blacks may face discrimination in the ability to purchase a house. Older households, for example, may also be *more* flexible in their responses to price and income changes if they follow Olsen's life-cycle model, or *less* flexible if they are subject to larger real or psychic moving costs. This section moves from the four 'economic' variables (permanent and transitory incomes, owner/renter price ratio, value-rent ratio) to the inclusion of the various combinations of the set of demographic variables (black, male household head, married household head, age of household head, household size).<sup>8</sup> Since the various models are nested within each other, the appropriate hypothesis test for the significance of  $(k_2 - k_1)$  additional parameters is  $(\ln L_2^* - \ln L_1^*) \sim (1/2)\chi_{(k_2 - k_1)}^2$ .

<sup>7</sup>The LIMDEP program is used for these estimates. Conversation with William Greene verifies that convergence problems are not uncommon in such estimation procedures. The renter demand results from this *semi*-maximum likelihood method are very similar to those obtained with an unconstrained (i.e. not estimated jointly with the owner demand) probit equation.

<sup>8</sup>Goodman and Kawai (1982) and others have shown that if current income is (incorrectly) used as a proxy for permanent income and transitory income,  $\alpha_T \leq \text{plim } a \leq \alpha_P$ , where  $\alpha_T$  and  $\alpha_P$  are the true coefficients of permanent income,  $Y^P$ , and transitory income,  $Y^T$ .

The data set is a random sample from the Annual Housing Survey for 1978. There are 1,951 observations, including 1,324 owners and 627 renters.<sup>9</sup> A permanent income regression was estimated using sociodemographic variables as instruments for the portion of income that can be treated as a return to human capital; interest and dividend payments were then added to the fitted value from the permanent income regression. Owner-occupied renter housing and renter housing prices (for standardized units), and value-rent ratios (for specific units) were calculated using hedonic price methods.<sup>10</sup>

Several specifications were tested for the probit (tenure choice) regression, and three were chosen for further discussion (raw regressions are available from the author on request). The *simple* model constrains  $\theta_1 = \theta_2 = \theta_3 = 0$ , and uses permanent income (*YP*) and transitory (*YT*) incomes, owner/renter price ratio (*RATIO*) and the value-rent ratio (*VALRENT*). All but *YT* are significant; of 1,951 observations, 378 or 19.4% are misclassified by tenure.

The *translated* model adds variables *MALE* (head of household), *AGE*, *MAR* (married status), *HHSIZE* (number of family members), and *BLACK*, thus allowing  $\theta_1$  to vary. *MALE*, *AGE*, and *MAR* are highly significant. All else equal, increased age and married status increase probability of ownership; male heads-of-household (holding family size constant), are less likely to own. *HHSIZE* and *BLACK* are not significant, although blacks are very slightly *more* likely to own. This positive, although statistically insignificant, result, with respect to race, holds through all formulations used, and is discussed further below. The addition of the demographic variables increases the log-likelihood function significantly (the difference is distributed  $\chi^2_5$ ). The number of misclassified observations falls to 318, or 16.3%, of all observations.

The *extended* model uses a Gorman transformation and then omits seven insignificant interactive variables. It is superior, according to the likelihood ratio criterion (there is further discussion of these tests below), to both the simple and the translated models, and reduces the number of misclassified observations to 309, or 15.8%.<sup>11</sup> The multicollinearity built into the extended model makes it difficult to evaluate individual variables, but subsequent calculations of elasticities away from variable means suggests important interactions that are not available with the simpler models.

Table 2 provides benchmark elasticities of tenure choice, evaluated at

<sup>9</sup>All house prices (for the hedonic estimates) and incomes (for the permanent income estimates) are deflated by cost of living deflators. In the demand estimates,  $P_o$  and  $P_r$  are formulated relative to the price of all other goods, following Polinsky (1977).

<sup>10</sup>Goodman (1988) discusses the derivations in detail.

<sup>11</sup>Race ('1' if the household was black, '0' otherwise) was kept in the model, even though it never tested significant in any way, no matter how it was entered. Omission of the seven insignificant interaction variables provides neither a significant decrease in the likelihood function nor any increase whatsoever in the number of bad predictions.



Table 2  
Elasticities at means from probit model.

	Simple (a)	Translated (b)	Extended (c)
YP	0.773	0.678	0.713
YT	-0.002	-0.002	
RATIO	-0.764	-0.956	-1.042
VALRENT	0.978	0.974	1.035
AGE		0.499	0.347
HHSIZE		0.046	0.074
BLACK		0.004	0.003
MALE		-0.111	-0.010
MAR		0.118	0.039
Predicted probability	0.775	0.787	0.785

variable means (see Appendix A, col. 1). Column (a) uses the simple model. The predicted probability of ownership, at the means, is 0.775. Income elasticity is 0.773, the relative price elasticity is -0.764, the value-rent elasticity is 0.978.<sup>12</sup> The latter two suggest, that a one percent increase in *both* the ratio of owner to renter prices *and* the value-rent ratio imply an elasticity of +0.214, or a positive impact on home ownership. This estimate is shown to be rather sensitive to the form of the estimating equation.

Columns (b) and (c) evaluate the translated and the extended models. The income, price and the value-rent ratio are fairly stable (across all three formulations). The demographic variables have differing effects depending on how they are introduced. Age, in particular, has a substantive and significant impact. A one percent increase in age implies a 0.499 (0.347) percent increase in the probability of owning in the translation (extended) model. Household size has a marginally positive (less than 0.1) elasticity in both formulations. Male head of household, and married status are more important in the translated model [column (b)] than in the extended model [column (c)].

Table 3.A shows the interactions of the relevant elasticities to the age of the household head (due, even in the translation case, to the interaction of the coefficients and the existing probability). As a result, simple translation shows the elasticity with respect to owner/renter relative price ratio (*RATIO*) declining from -1.611, for 25 year olds, to -0.323 for those aged 65. Income elasticities fall from 1.144 to 0.229. At the mean age, income elasticity is slightly lower than for the simple model, and the combined relative price/value-rent elasticity is also lower. The extended model provides more

<sup>12</sup>Elasticities are estimated using incremental probabilities of cumulative density function implied by the index from the probit analysis. In the binomial probit formulation, they will be similar, although not analytically identical to the analogous binomial logit model which generates own-elasticity of  $\beta_i x_i (1-P)$ , with  $P$  referring to the probability.

Table 3  
Probit probabilities and elasticities away from the means.

A. Elasticities by age						
Age	Pred prob	E age	E income	E ratio	E V	E rat: V
Translation						
25	0.603	0.513	1.144	-1.611	1.642	0.031
45	0.817	0.469	0.580	-0.817	0.833	0.016
65	0.939	0.268	0.229	-0.323	0.329	0.006
Extended (trans/scaled) estimate						
25	0.665	0.312	1.290	-2.223	1.970	-0.254
45	0.811	0.342	0.602	-0.826	0.858	0.032
65	0.909	0.266	0.255	-0.160	0.278	0.118
B. Elasticities by income						
Income	Pred prob	E age	E income	E ratio	E V	E rat: V
Simple						
10.8	0.442	-	0.977	-1.761	2.255	0.494
28.6	0.951	-	0.309	-0.210	0.269	0.059
Translation						
10.8	0.486	1.079	0.805	-2.069	2.109	0.040
28.6	0.943	0.159	0.314	-0.305	0.311	0.006
Extended (trans/scaled) estimate						
10.8	0.469	1.222	0.890	-2.374	2.358	-0.016
28.6	0.951	0.045	0.298	-0.301	0.299	-0.002

flexibility still. All elasticities (and particularly the ratio of owner to rental prices) are more responsive to age than above.

Table 3.B allows income to vary. Mayo (1981) and others have shown how the reference level of income can imply major differences in income elasticities in housing demand studies.<sup>13</sup> It is apparent that this can occur in tenure choice decisions as well. Compare the parameters estimated from the simple model to the extended model. Reference incomes are mean income (\$19,711) less one standard deviation (\$10,804), and plus one standard deviation (\$28,618). Two conclusions are important here. First, the simple model tends to overstate the tenure choice income elasticity for low-income households, relative to either the translation, or to the extended estimate. Second, the simple model overstates the contribution of the value-rent effect,

<sup>13</sup>Consider the general Box-Cox equation,

$$(Q^{\lambda_1} - 1)/\lambda_1 = \alpha(Y^{\lambda_2} - 1)/\lambda_2.$$

Goodman and Kawai (1986) derive generalized 'second elasticities' (elasticity of the income [and/or price] elasticity with respect to income [and/or price]). The second elasticity for income equals  $\lambda_2 - \lambda_1 \eta_y$ . In the standard linear form (log-log) with  $\lambda_1 = \lambda_2 = 1$  or  $(\lambda_1 = \lambda_2 = 0)$ , this implies that elasticities rise with income (are constrained to be constant).

Table 4  
Joint likelihood estimates for tenure choice and demand.

Demand	Tenure choice		
	Simple	Translation	Extended
Simple	-2656.58	-2581.23 <sup>b</sup>	-2553.61 <sup>b</sup>
Translation	-2641.03 <sup>a</sup>	-2555.41 <sup>a, b</sup>	-2528.47 <sup>a, b</sup>
Extended	-2614.51 <sup>a</sup>	-2529.17 <sup>a, b</sup>	-2503.44 <sup>a, b</sup>

<sup>a</sup>Significantly different from previous row.

<sup>b</sup>Significantly different from previous column.

compared to the relative housing price ratio. For those with renter incomes a 1% increase in both the price ratio and the value-rent ratio, implies a 0.494% increase in the probability of owning, using the simple model. This compares to estimates of 0.040 and -0.016, with the translation and the extended models.

#### 4.2. Joint likelihood maximization

In the discussion above, (7b) provides a format for maximizing the joint likelihood function of the tenure choice decision and the two demand equations. This section maximizes this function for a set of three separate probit regressions, three linear rental regressions and three linear owner regressions. In all cases, removal of restrictions [i.e. specification of the interaction and/or translation terms from eq. (1)] leads to significant changes (at the 5% level or better) in the likelihood functions. The 'best' combination (in italics in table 4) uses extended formulations for both the tenure choice and the demand regressions. This suggests that even though many analysts routinely use simple translation techniques, more detailed inclusion of interactive terms is indicated in both the tenure choice, and the demand regressions.

#### 4.3. Demand estimates

This section examines both the partial and the full demand elasticities estimated at the means, for the different housing tenures. Using the extended probit regression as a conditioning device, the three linear regressions discussed above are estimated. Table 5 provides the resulting elasticities (the regressions themselves are available on request) at the means. Later discussion (table 6) considers the calculation of elasticities *away* from the means. Accurate estimation of such parameters may have substantive implications for various policy actions which affect special groups (most often the poor), who may face lower incomes, and/or higher housing prices.

Table 5  
Partial and full elasticities.

A. Tenure choice elasticities				
	Reference	Simple	Translated	Extended
Y	19.7110	0.620	0.508	0.516
RATIO	152.9266	-0.555	-0.648	-0.683
VALRENT	152.4200	0.785	0.729	0.748
AGE	40.9300		0.373	0.169
BLACK	0.0836		0.003	0.002
MALE	0.8288		-0.083	-0.007
MAR	0.7135		0.088	0.028
HHSIZE	3.0410		0.035	0.054

  

B. Partial renter and owner demand elasticities							
Renter elasticities			Owner elasticities				
	Simple (R.1)	Translated (R.2)	Extended (R.3)		Simple (O.1)	Translated (O.2)	Extended (O.3)
$Q_R$	1.200	1.244	1.254	$Q_O$	1.534	1.562	1.538
Elas. $Y_R$	0.248	0.223	0.235	Elas. $Y_O$	0.269	0.262	0.249
Elas. $P_R$	-0.056	-0.142	-0.107	Elas. $P_O$	-0.515	-0.561	-0.524
Elas. $V_R$	—	—	—	Elas. $V_O$	0.621	0.664	0.619
Elas. $A_R$	—	-0.225	-0.197	Elas. $A_O$	—	-0.194	-0.193
Elas. $HH_R$	—	0.003	-0.012	Elas. $HH_O$	—	-0.026	-0.022

  

C. Total elasticities			
	Simple	Translated	Extended
$H(Q)$	1.476	1.513	1.497
Elas. $Y^*$	0.382	0.347	0.330
Elas. $P_{R^*}$	0.096	0.097	0.097
Elas. $P_{O^*}$	-0.546	-0.604	-0.570
Elas. $P^*$	-0.450	-0.507	-0.473
Elas. $V^*$	0.680	0.709	0.664
Elas. $P:V^*$	0.230	0.201	0.191
Elas. $A^*$	0.086	-0.053	-0.092
Elas. $HH^*$	—	-0.016	-0.010

Several points should be made in comparing elasticities for owners and renters. As noted above, if linear forms are used, and the income elasticity is less than +1.0, then increased incomes lead to increased measured elasticities.<sup>14</sup> Goodman and Kawai (1984b) control for income and find that the different elasticities reported for owners and renters largely disappear. Similarly, higher reference incomes lead to lower measured price elasticities. Again, this is controlled here.

Although much of the recent literature suggests the use of permanent rather than current income for demand relationships, Goodman (1988) shows that after conditioning in the probit equation, there is no significant difference in the income coefficients of the demand estimates (although there is significant difference in the probit results). As a result, in this paper, permanent and transitory incomes are combined, and current income is used for the demand regressions.

Variable means are used to evaluate the owner and renter elasticities (see appendix A, cols. 2 and 3), conditional on the tenure choice (elasticities displayed in table 5.A).<sup>15</sup> Since subsequent analysis will compare different demographic categories, it is appropriate to use integer values. Renter and owner housing are discussed in table 5.B. The three renter equations can be treated first. Simple demand (eq. R.1) and translated demand (R.2) are nested within the extended case (R.3) and the *F*-tests for the added variables are significant. Income elasticity is almost invariant among the three specifications (varying from 0.223 to 0.248). Price elasticity varies slightly more, from -0.056 in R.1, to -0.142 in R.2 and falling back to -0.107 in R.3. Conditional on tenure choice, older households purchase less rental housing. The sample selection parameter  $\lambda_r$  (used due to failure to maximize the joint likelihood function over the renter equation) shows significant sample selection impacts in all cases (with respect to renter housing demand).

For owner housing, denoting the three equations (as above) O.1, O.2 and O.3, the income elasticity falls slightly when the extended model is used (the three equations yield 0.269, 0.262, and 0.249, respectively). Price elasticity, even with the added parameters allowing for more curvature, is largely invariant (-0.515, -0.561, and -0.524, respectively).<sup>16</sup> Simultaneous one percent increases in both the price (*PO*) and the value-rent ratios (*VALRENT*) show positive impacts. As with renters, owner age is negatively

<sup>14</sup>Other work [for example, Goodman and Kawai (1986)] has used more flexible functional forms to address this problem. Repeated attempts to use even semi-logarithmic forms in this paper led to extremely high correlations between error terms in the conditioning probit equation and in the subsequent demand equations.

<sup>15</sup>The columns differ slightly because the mean housing price divided by the mean rent is not identical to the mean *RATIO*.

<sup>16</sup>Many attempts to add terms interacting with *AGE* led to collinearity problems within the equation, and between the equation and the probit conditioning equation.

related to demand, and the elasticities are almost the same (approximately  $-0.2$ ) across R.2, R.3, O.2 and O.3.

The estimated coefficients from the probit, owner and renter equations are then used to calculate the total elasticities at the means (presented in table 5.C). Since the extended model is preferable according to likelihood ratio criteria, subsequent discussion refers to it. Total estimated demand  $H(Q)$  is slightly higher with the extended model than with the simple or the translation model. Full income elasticity  $\eta_y^*$  is 0.330, and full housing price elasticity  $\eta_p^*$  (raising both owner and renter prices by 1%) is  $-0.473$ . An increase in owner price (holding renter price constant) both decreases the amount of owner housing, and shifts demand toward (smaller) renter housing, yielding a full owner price elasticity  $\eta_{p_o}^*$  of  $-0.570$ . An increase in renter price, on the other hand, can have a positive effect on  $H(Q)$ , with the negative impact on renter quantity offset by the switch to (larger) owner housing. The full renter price elasticity  $\eta_{p_r}^*$  is positive, and largely invariant to the estimating equation used. A one percent increase in the price of housing and the value-rent ratio implies a net positive impact ( $+0.191$ ). Age, although positively related to ownership, is negatively related (as noted above) to demand, for both owners and renters; the full age elasticity is  $-0.092$ . Household size is slightly negatively related to quantity.<sup>17</sup>

#### 4.4. *Demographics and full elasticities*

Although linear demand systems imply higher elasticities with higher income levels, a thoughtful examination of (2) suggests no easy analytic solution. Elasticities of the probability functions are highest at a probability of 0.5 and the interactive terms in the two demand equations suggest that variations of the relevant elasticities may be non-linear and perhaps non-monotonic.

Full elasticities can be compared across several demographic dimensions. Consider, for example, a household with head aged 27 years (mean age minus one standard deviation), married status, and one additional member (presumably a child). One can trace income and price elasticities, housing demand  $H(Q)$ , and the probability of ownership across mean incomes and owner prices, and the same incomes and owner prices plus/minus one standard deviation. One can do the same for the comparable household with mean age (41) and the same demographic composition.<sup>18</sup>

Whites have higher income elasticities than blacks at both ages, and over

<sup>17</sup>In an analysis of individuals, elasticities with respect to qualitative demographic variables are probably not meaningful.

<sup>18</sup>Many other simulations have been constructed. They are available from the author on request.

all price-income pairs. For example, mean white (black) income elasticity (mean population income) at age 27 (one standard deviation below the mean) is 0.383 (0.307); at age 41 the mean white (black) income elasticity is 0.334 (0.293). The disparity is generally highest for those facing the highest housing prices (usually central cities), where white elasticities can be as much as 75% higher. Income elasticities for both blacks and whites increase monotonically with respect to income, and elasticities evaluated at low incomes are often less than half those evaluated at the mean income.

Price elasticities indicate slightly different patterns. In higher-priced, lower-income cells, blacks have lower price elasticities than whites; the elasticities reverse for lower-priced, higher-income cells. Older households have more price elastic demand than do younger ones in all cells, but particularly at higher prices.

Both the income and the price elasticities provide guidance on predicting the performance (with respect to increased housing purchase) of market-based housing programs which depend on income or rent subsidies to increase demand. From the findings presented here, it would appear that such programs are concentrating on the groups that have the least responsive demands for housing; i.e., the young, the poor, those in high cost (urban) housing, and minorities.

Two other results merit discussion. First, no matter how income, price, and age are controlled, white demand  $H(Q)$ , is significantly larger than black demand. Define relative housing demand  $R = H^w/H^b$ . For a group age 27 (age 41), across incomes and owner prices that are varied from one standard deviation below to one standard deviation above their means,  $R$  ranges from 1.045 to 1.249 (1.154 to 1.266). In a sample that was selected well over a decade after the advent of far-reaching open-housing legislation, it is hard to argue racial discrimination, although Yinger (1986) suggests that it may still be pervasive. On the other hand, the flexible forms used should handle much of what is generally considered to constitute 'tastes'.

$R$  varies with the relevant full demand elasticities. Using income, for example  $(dR/R)/(dY/Y) \cong 0$  as  $(\eta_y^{*w}/\eta_y^{*b}) \cong R$ . Although blacks are actually slightly more likely to own than whites (in fact, the increased probabilities of black ownership are most pronounced under the least favorable circumstances, that is, the lowest incomes, and the highest prices), they purchase relatively less housing than do whites.<sup>19</sup> Consider black and white households (head aged 27) subject to identical (mean) permanent incomes, prices, and demographics. Whites are 6.2% less likely to own, but as renters they

<sup>19</sup>Separate probit equations were also run for both white and blacks and were not significantly different from the single equation results by likelihood ratio criteria. At lower incomes and higher prices, blacks were slightly more likely to own than whites. At mean, or white incomes and lower prices, whites were insignificantly more likely to own (probabilities differing by less than 0.01).

purchase 8.2% more housing, and as owners they purchase 26.8% more housing, leading to  $R=1.212$ . Since  $(\eta_y^{*W}/\eta_y^{*B})=0.383/0.307=1.247$ , increased incomes lead to increased  $R$ . This analysis still does not indicate *why* whites purchase more housing. The differential can be explained by different black tastes for housing, but it is also consistent with reduced housing opportunities for (often larger) owner-occupied units, possibly related to discrimination.

The second result refers to the effects of demographic variables on the impacts of economic variables. Eq. (6f) shows that if permanent income is related to age  $A$ , for example, then the owner and renter demand, and the tenure choice all have both *direct* (age alone) and *indirect* (age on income, and income on the outcome) components. Columns 2 and 3 of both the partial and the total elasticities of table 5 show negative values for  $\eta_A^*$ .

The simple linear demand model suggests that these negative age elasticities should lead to *increased* income elasticities as age rises. By definition,  $\eta_Y=(dQ/dY)(Y/Q)$ ;  $\eta_P=(dQ/dP)(P/Q)$ . In a nonlogarithmic formulation (with logarithms, all elasticities are constant), negative age elasticities imply a decreased reference level for  $Q$ , and hence a higher income elasticity for a given income level and estimated income coefficient. Such cross-elasticities can be referred to as *second elasticities* [following Goodman and Kawai (1984a)]. They are derived by fully differentiating eqs. (6a)–(6e) for income, housing price, and value-rent ratio, with respect to age  $A$ :

$$\begin{aligned} \eta_{YA}^* &= E_{oy}[\eta_{fA^*} + \eta_{QoA^*} + \eta_{QoyA} - \eta_A^*] + E_{ry}[-\eta_{fA^*} + \eta_{QrA^*} + \eta_{QryA} - \eta_A^*] \\ &\quad + E_{fy}[(q_o\eta_{QoA^*} - q_r\eta_{QrA^*}) + (q_o - q_r)(\eta_{fyA} - \eta_A^*)], \end{aligned} \quad (8a)$$

$$\begin{aligned} \eta_{PA}^* &= E_{op}[\eta_{fA^*} + \eta_{QoA^*} + \eta_{QopA} - \eta_A^*] + E_{rp}[-\eta_{fA^*} + \eta_{QrA^*} + \eta_{QrpA} - \eta_A^*], \end{aligned} \quad (8b)$$

$$\begin{aligned} \eta_{VA}^* &= E_{ov}[\eta_{fA^*} + \eta_{QoA^*} + \eta_{QovA} - \eta_A^*] \\ &\quad + E_{fv}[(q_o\eta_{QoA^*} - q_r\eta_{QrA^*}) + (q_o - q_r)(\eta_{fvA} - \eta_A^*)], \end{aligned} \quad (8c)$$

where  $E_{oi}=(fq_o/A)\eta_{Qoi}$ ,  $E_{ri}=((1-f)q_r/A)\eta_{Qri}$ , and  $E_{fi}=\eta_{fii}/A$ , with  $i$  referring to economic variables income, price and value-rent. Asterisks refer to *full* effects of demographic variables (both direct and indirect) as noted in (6f).  $E_{oi}$ ,  $E_{ri}$ , and  $E_{fi}$  are elasticities which are weighted by the probability (or percentage) owner (renter) and quantity of housing consumed per owner (renter). Terms in square brackets show how each is modified by changes in age (or other demographic variables). Eq. (8a) has all three terms since income enters all three decisions (tenure choice, owner and renter demand). Eq. (8b) omits the tenure choice term since tenure choice is homogeneous in



relative owner and renter prices. Eq. (8c) omits the direct renter effect, since value-rent ratio does not affect renter housing consumption.

Determining the signs of these second elasticities is an empirical matter. While age is negatively related to owner and renter quantity demanded, it is positively related to probability of owner tenure (more housing), which leads to increased demand. Further, it interacts with the price and income variables in all three (the two demand and the tenure choice) equations.

Table 6 shows this effect. Consider income elasticities  $\eta_{Q_o,y}$  or  $\eta_{Q_r,y}$ . Consistent with Olsen, these elasticities, conditional upon tenure choice, rise with age. The full income elasticity  $\eta_y^*$ , however, may fall. If increased age leads to a shift to owner housing, and hence increased housing quantity, the quantity increase due to tenure choice, offsets the tenure-specific decreases due to age. Since a general housing price increase (one percent increases for both owners and renters) does not affect tenure choice, housing consumption does not increase due to change in tenure, and total price elasticity  $\eta_P^*$  may rise (away from zero) with age.

## 5. Conclusions

This research has attempted to examine the impacts of several demographic variables on housing demand in a systematic manner. It has differed from other studies that examine budget shares or household level housing demand, in its detailed examination of the interactions at several levels of demand.

Age and race have particular impacts. *Ceteris paribus*, older households are more likely to own, but demand by both owners and renters is inversely related to age. Controlling for the numerous interactions, and differential incomes and prices faced, blacks do purchase less housing than whites. Both income and price elasticities vary across the many age-race-price-income combinations. Household size may matter, but not very much.

The research suggests that one fruitful avenue of inquiry must be in the determination of 'tastes'. Are they race-related, and do they vary with age, and why? Although economists usually leave this topic to others, it is clear that further work examining tastes would have salutary results in the understanding of housing demand.

Table 6  
Income, price, and value-rent elasticities by age.

Age	Renter elasticities		Owner elasticities				Full elasticities			
	Elas. $Y_R$	Elas. $P_R$	Elas. $Y_O$	Elas. $P_O$	Elas. $V$	Elas. $P_O:V$	Elas. $Y^*$	Elas. $P^*$	Elas. $V^*$	Elas. $P:V^*$
20	0.204	-0.300	0.226	-0.477	0.563	0.086	0.391	-0.443	0.715	0.272
25	0.211	-0.256	0.231	-0.488	0.576	0.088	0.373	-0.447	0.697	0.250
30	0.218	-0.211	0.237	-0.498	0.588	0.090	0.357	-0.453	0.682	0.229
35	0.226	-0.164	0.242	-0.510	0.602	0.092	0.344	-0.461	0.672	0.211
40	0.234	-0.116	0.248	-0.522	0.616	0.094	0.332	-0.471	0.665	0.194
45	0.241	-0.067	0.254	-0.534	0.631	0.097	0.323	-0.482	0.661	0.179
50	0.250	-0.016	0.260	-0.548	0.646	0.098	0.316	-0.495	0.662	0.167
55	0.258	0.037	0.266	-0.561	0.663	0.102	0.311	-0.509	0.666	0.157
60	0.266	0.091	0.273	-0.576	0.680	0.104	0.308	-0.525	0.673	0.148

## Appendix A

Table A.1  
List of variables.

<i>YP</i>	Permanent income
<i>YT</i>	Transitory income
<i>Y</i>	Current income
$P_o$	Owner-occupied housing price
$P_r$	Renter housing price
<i>RATIO</i>	$P_o/P_r$
<i>VALRENT (V)</i>	Value-rent ratio
<i>BLACK</i>	'1' if black household head; '0' otherwise
<i>MALE</i>	'1' if male household head; '0' otherwise
<i>MAR</i>	'1' if married household head; '0' otherwise
<i>AGE (A)</i>	Age of household head
<i>HHSIZE (HH)</i>	Household size

Table A.2  
Bundles to evaluate probability and demand.

	Probability	Owner demand	Renter demand
<i>Y</i>	19.71	19.71	19.71
<i>RATIO/PRICE</i>	168.74	33,342	217.42
<i>VALRENT</i>	152.42	152.42	152.42
<i>AGE</i>	40.93	40.93	40.93
<i>BLACK</i>	0.08355	0.0	0.0
<i>MALE</i>	0.82881	1.0	1.0
<i>MAR</i>	0.71348	1.0	1.0
<i>HHSIZE</i>	3.041	3.0	3.0

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