

just to make its arguments explicit.

$\mathcal{D}(\mu, \rho) = \{D_h(X | \mu, \rho) : h \in \mathcal{H}^F\}$: all possible full data functions obtained by varying h , but fixing μ, ρ .

$\mathcal{D} = \{D_h(X | \mu(F_X), \rho(F_X)) : F_X \in \mathcal{M}^F, h \in \mathcal{H}^F\}$: all possibly full data structure estimating functions obtained by varying h and F_X .

$S_{\text{eff}}^{*F}(\cdot | \mathbf{F}_X)$: the canonical gradient (also called efficient influence curve) of the pathwise derivative of the parameter $\mu(F_X)$ in the full data model \mathcal{M}^F .

$\mathbf{T}_{\text{nuis}}^{F, \perp, *}(F_X)$: the set of all gradients of the pathwise derivative at F_X of the parameter $\mu(F_X)$ in the full data model \mathcal{M}^F whose components span $T_{\text{nuis}}^{F, \perp}(F_X)$.

$D_{\text{heff}}(\cdot | \mu(F_X), \rho(F_X)) = S_{\text{eff}}^{*F}(\cdot | F_X)$: that is, h_{eff} indexes the optimal estimating function in the *full data structure model*. Here $h_{\text{eff}} = h_{\text{eff}}(F_X)$ depends on F_X . Of course, one still obtains an optimal estimating function by putting a $k \times k$ fixed matrix in front of S_{eff}^{*F} .

$\mathcal{F}(t)$: a predictable observed subject-specific history up to time t , typically representing all observed data up to time point t on a subject.

A : a time-dependent possibly multivariate process $A(t) = (A_1(t), \dots, A_k(t))$ whose components describe specific censoring (e.g., treatment) actions at time t . Here A represents the censoring variable C for the observed data structure. Typically, $A_j(t)$, $j = 1, \dots, k$, are counting processes.

A : the support of the marginal distribution of A .

$\alpha(t) = E(dA(t) | \mathcal{F}(t))$: the intensity (possibly discrete, $\alpha(t) = P(dA(t) = 1 | \mathcal{F}(t))$ at given grid points) of counting process $A(t)$ w.r.t. the history $\mathcal{F}(t)$.

$\mathbf{Y} = (\mathbf{A}, \mathbf{X}_A)$: a particular type of observed censored data, where for the full data we have $X = (X_a : a \in A)$, and A tells us what component of X we observe. For example, $A(t)$ can be the indicator $I(C \leq t)$ of being right-censored by a dropout time C . If the full data model is a causal model and there is no censoring, then $A(t)$ is the treatment that the subject receives at time t . If the observed data structure includes both treatment assignment and censoring, then $A(t)$ is the multivariate process describing the treatment actions and censoring actions assigned to the subject at time t .

$L_0^2(\mathbf{P}_{F_X, G})$: Hilbert space of functions $V(Y)$ with $E_{P_{F_X, G}} V(Y) = 0$ with inner-product $\langle h, g \rangle_{P_{F_X, G}} = E_{P_{F_X, G}} h(Y)g(Y)$ and corresponding norm $\|h\|_{P_{F_X, G}} = \sqrt{E_{P_{F_X, G}} h^2(Y)}$.

$\mathbf{T}(\mathbf{P}_{F_X, G}) \subset L_0^2(P_{F_X, G})$, $\mathbf{T}_{\text{nuis}}(\mathbf{P}_{F_X, G}) \subset L_0^2(P_{F_X, G})$, $\mathbf{T}_{\text{nuis}}^\perp(\mathbf{P}_{F_X, G}) \subset L_0^2(P_{F_X, G})$ are the observed data tangent space, observed data nuisance tangent space, and the orthogonal complement of the observed data nuisance tangent space at $P_{F_X, G}$, respectively, in model $\mathcal{M}(CAR)$ (or, if made explicit in $\mathcal{M}(G)$, where μ is the parameter of interest.

$\mathbf{T}_{CAR}(\mathbf{P}_{F_X, G}) = \{V(Y) : E_G(V(Y) | X) = 0\} \subset L_0^2(P_{F_X, G})$: the nuisance

tangent space of G in model $\mathcal{M}(CAR)$.

$\mathbf{T}_2(\mathbf{P}_{F_X, G}) \subset T_{CAR}(P_{F_X, G})$ or $\mathbf{T}_G(\mathbf{P}_{F_X, G}) \subset T_{CAR}(P_{F_X, G})$: the nuisance tangent space of G in the observed data model $\mathcal{M}(G)$.

$D \rightarrow \mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}_0, \mathbf{G}, \mathbf{D})$, $D \rightarrow \mathbf{IC}(\mathbf{Y} | \mathbf{F}_X, \mathbf{G}, \mathbf{D})$, $D \rightarrow \mathbf{IC}(\mathbf{Y} | \mathbf{Q}, \mathbf{G}, \mathbf{D})$: mapping from a full data function into an observed data function indexed by nuisance parameters $Q_0(F_X, G)$, G , F_X, G or $Q(F_X, G)$, G . $\mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}_0, \mathbf{G}, \mathbf{D})$ stands for an initial mapping and $\mathbf{IC}(\mathbf{Y} | \mathbf{F}_X, \mathbf{G}, \mathbf{D})$ and $\mathbf{IC}(\mathbf{Y} | \mathbf{Q}(\mathbf{F}_X, \mathbf{G}), \mathbf{G}, \mathbf{D})$ for the optimal mapping orthogonalized w.r.t. T_{CAR} or a mapping orthogonalized w.r.t. a subspace of T_{CAR} . In many cases, it is not convenient to parametrize \mathbf{IC} in terms of F_X, G , but instead parametrize it by a parameter $Q = Q(F_X, G)$ and G . We note that the dependence of these functions on F_X and G is only through the F_X -part of the density of Y and the conditional distribution of Y , given X , respectively.

The mapping \mathbf{IC}_0 satisfies for each $P_{F_X, G} \in \mathcal{M}(G)$: for a non empty set of full data functions $\mathcal{D}(\rho_1(F_X), G)$, we have

$$E_G(\mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}, \mathbf{G}, \mathbf{D}) | X) = D(X) \text{ } F_X\text{-a.e. for all } Q \in \mathcal{Q}_0. \quad (1)$$

For \mathbf{IC} , we have the additional property at each $P_{F_X, G} \in \mathcal{M}(CAR)$:

$$\begin{aligned} \mathbf{IC}(\mathbf{Y} | \mathbf{Q}(\mathbf{F}_X, \mathbf{G}), \mathbf{G}, \mathbf{D}) &= \mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}_0(\mathbf{F}_X, \mathbf{G}), \mathbf{G}, \mathbf{D}) \\ &\quad - \Pi_{F_X, G}(\mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}_0(\mathbf{F}_X, \mathbf{G}), \mathbf{G}, \mathbf{D}) | T_{CAR}), \end{aligned}$$

or the projection term can be a projection on a subspace of T_{CAR} . Here $\Pi(\cdot | T_{CAR})$ denotes the projection operator in the Hilbert space $L_0^2(P_{F_X, G})$ with inner product $\langle f, g \rangle_{P_{F_X, G}} = E_{P_{F_X, G}} f(Y)g(Y)$.

$\mathcal{D}(\rho_1(\mathbf{F}_X), \mathbf{G})$: the set of full data functions in \mathcal{D} for which (1) holds. Thus, these are the full data structure functions that are mapped by \mathbf{IC}_0 into unbiased observed data estimating functions. By making the appropriate assumption on the censoring mechanism, one will have that $\mathcal{D}(\rho_1(\mathbf{F}_X), \mathbf{G}) = \mathcal{D}$, but one can also decide to make this membership requirement $D_h(\cdot | \mu(F_X), \rho(F_X)) \in \mathcal{D}(\rho_1(\mathbf{F}_X), \mathbf{G})$ a nuisance parameter of the full data structure estimating function: see next entry.

$D_h(\cdot | \mu(\mathbf{F}_X), \rho(\mathbf{F}_X, \mathbf{G}))$, $h \in \mathcal{H}^F$: these are full data structure estimating functions satisfying $D_h(\cdot | \mu(F_X), \rho(F_X, G)) \in \mathcal{D}(\rho_1(\mathbf{F}_X), \mathbf{G})$ for all $h \in \mathcal{H}^F$. Formally, they are defined in terms of initially defined full data estimating functions D_h as

$$D_h^r(\cdot | \mu, \rho, \rho_1, G) \equiv D_{\Pi(h | \mathcal{H}^F(\mu, \rho, \rho_1, G))}(\cdot | \mu, \rho),$$

where $\mathcal{H}^F(\mu, \rho, \rho_1, G) \subset \mathcal{H}^F$ are the indexes that guarantee that $E_G(\mathbf{IC}_0(\mathbf{Y} | \mathbf{Q}_0, \mathbf{G}, D_h(\cdot | \mu, \rho)) | X) = D_h(X | \mu, \rho) \text{ } F_X\text{-a.e}$ and $\Pi(| \mathcal{H}^F(\mu, \rho, \rho_1, G))$ is a mapping from \mathcal{H}^F into $\mathcal{H}^F(\mu, \rho, \rho_1, G)$ that is the identity mapping on $\mathcal{H}^F(\mu, \rho, \rho_1, G)$. Thus, if $D_h(\cdot | \mu(F_X), \rho(F_X)) \in \mathcal{D}(\rho_1(\mathbf{F}_X), \mathbf{G})$ for all $P_{F_X, G} \in \mathcal{M}(G)$, then $D_h^r = D_h$. For notational convenience, we denote $D_h^r(\cdot | \mu, \rho, \rho_1, G)$ with $D_h(\cdot | \mu, \rho)$ again, but where ρ