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Using Lorenz Curves to Characterise Urban Elderly Populations

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Summary. This study measures urban elderly distributions using Lorenz curves and Gini coefficients estimated from 1980 Census data. The results suggest ways that such summary measures can be used to examine population distributions among urban areas. The paper considers three metropolitan areas, Baltimore, Philadelphia, and Pittsburgh. The elderly are more concentrated in the central cities of Philadelphia and Baltimore than in Pittsburgh, even though the Pittsburgh SMSA has the largest elderly percentage of the three. The elderly and the poor elderly are more concentrated in Baltimore than in Philadelphia, and both are more concentrated than in Pittsburgh.

Introduction

Although the problems of an ageing population are not exclusively urban, they are important within urban areas. Older Americans comprise disproportionately large shares of central city (relative to metropolitan) populations. To the extent that the elderly are large users of urban services, the central cities may bear disproportionate costs for their support.

This study measures urban elderly distributions using Lorenz curves, quantifying each with the Gini coefficient, and estimated from 1980 Census data. The results suggest ways that such summary measures can be used to examine population distributions among urban areas.

Three metropolitan areas, Baltimore, Philadelphia, and Pittsburgh, were examined, all are located in the Mid-Atlantic region and were developed at approximately the same time. Although a sample of three obviously does not permit broad generalisation, the three areas do provide a comparative perspective.

In the resulting analyses, Philadelphia and Baltimore are generally quite similar, but Pittsburgh differs. The elderly are more concentrated in the central cities of Philadelphia and Baltimore than in Pittsburgh, even though the Pittsburgh SMSA has the largest elderly percentage of the three. The elderly and the poor elderly are more concentrated in Baltimore than in Philadelphia, and both are more concentrated than in Pittsburgh.

Locations of the Elderly — Descriptive Theory

It is useful to examine the age distribution of urban residents in the context of urban location models.[1] Simple versions of such models say little about the ages of residents; they might suggest that in the long run, the elderly (or any other age group) should be distributed similarly, in any particular neighbourhood, to their distribution in the general population. That is, if the elderly comprise 10 per cent of a metropolitan area's population, they should comprise 10 per cent of each neighbourhood.

One should be cautious about such inferences. As cities develop, neighbourhoods are built and develop at specific times, attracting residents with similar

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[1] Such models (McDonald, 1979) have considered the age distribution of structures.

ages, family sizes (either current or expected), or housing preferences (large/small, single family/multiple dwelling). Neighbourhoods may develop and age together, and may come to be defined by residents' ages. Residents, although originally drawn by housing type or size, may appreciate the neighborhood itself and change residences within it.

There are other reasons why older people may gather in specific neighbourhoods. Apart from age-segregated housing, hospitals, or nursing homes, certain housing types may cater explicitly to older households. Those who no longer need large dwelling units may prefer smaller ones (which are geographically limited), often with limitations on children (which leads to still further concentration). People with limited incomes may be forced to rent smaller units, or in locations with inexpensive rental housing (which again may impose geographic limitations).

Lorenz Curves and Gini Coefficients

Lorenz curves can be adapted to quantify these variations in elderly distributions more precisely by ranking neighbourhood-level observations of elderly populations from smallest to largest. The analysis then indicates that the lowest ranked (in terms of elderly population) 20 per cent of the census tracts, for example, had 5 per cent of the elderly, the next 20 per cent had an additional 8 per cent of the elderly (or alternatively that the lowest 40 per cent had 13 per cent).

Gini coefficients (henceforth, Ginis) are often used to summarise Lorenz curves. Consider the Lorenz curve in which the cumulative percentage of tracts is plotted against the cumulative percentage of elderly. The Gini compares the area L, between the diagonal (signifying equal distribution) and the Lorenz curve, to the entire area under the diagonal, T. As L/T approaches 0 (1) the population under study is more (less) equally distributed.

More formally, consider the cumulative percentage of tracts, F, and the cumulative percentage of elderly, F_1. Define two parameters, η, and π, where:

$$\eta = (F - F_1)/\sqrt{2}, \tag{1a}$$

and:

$$\pi = (F + F_1)/\sqrt{2}. \tag{1b}$$

The function defined by Kakwani and Podder (1976) is:

$$\eta = a\pi^{\alpha}(\sqrt{2} - \pi)^{\beta}, \tag{2}$$

or in logs,

$$\log(\eta) = \log(a) + \alpha \log \pi + \beta \log(\sqrt{2} - \pi). \tag{2'}$$

Equation (2') is estimated with least squares methods. Gini G is:

$$G = 2^{(3+\alpha+\beta)/2} aB(1 + \alpha, 1 + \beta), \tag{3}$$

where B is the statistical Beta function.

Several observations should be made. Rasche *et al.* (1980) and Rossi (1985) note that the function is not well-behaved at its endpoints; slopes at the limits are not zero and infinite, as would be theoretically desirable. On the other hand, it is easily estimated (alternative forms require nonlinear iterative methods), and the calculated Ginis are not sensitive to the endpoint problems. Kennedy (1979) notes that least squares estimates of a in logarithms, as in (2'), will be biased upward. His adjustment, involving the variance of the estimated parameter, is used here.

The two sets of Lorenz curves presented here examine elderly population, and elderly population in poverty. Data at the Census tract level do not permit extensive cross-tabulation by age but they do disaggregate the elderly population by poverty classification. Poverty may be particularly burdensome for the elderly since they may have more health problems than others, or greater difficulties mitigating their monetary problems by working. We hypothesise that the poor elderly, like many of the poor, are more concentrated than the elderly population as a whole.

Baltimore, Philadelphia and Pittsburgh

The methods are applied to three Mid-Atlantic metropolitan areas. Baltimore, Philadelphia and Pittsburgh serve as central cities to their surrounding areas. Philadelphia and Pittsburgh both contain higher concentrations of the elderly than does Baltimore. In Philadelphia and Pittsburgh, 14.3 and 16.0 per cent of the population, respectively, is age 65 and over (compared to 12.9 per cent for Pennsylvania); both compare to 12.7 per cent in Baltimore (9.6 per cent for Maryland).

In both the Baltimore and the Philadelphia SMSAs, the largest concentrations of the elderly are in the central cities. For description, it is helpful to

Table 1

Ten tracts with largest elderly populations

City	Number of tracts	Percent tracts in ten highest	Elderly population ten highest	Total elderly population	Per cent in ten highest
Baltimore	662	1.5	15,936	219,107	7.3
Philadelphia	948	1.1	23,230	456,549	5.1
Pittsburgh	773	1.3	12,530	301,324	4.2

locate the 10 tracts in each area with the largest elderly populations. Seven of these 10 tracts in the Baltimore SMSA are in Baltimore City. These tracts, comprising approximately 1.5 per cent of the 662 tracts in the SMSA, have 15,936 elderly residents (or 7.3 per cent of the SMSA elderly population, as noted in Table 1). In the Philadelphia SMSA, all 10 tracts are in the City of Philadelphia (and somewhat concentrated in Northeast Philadelphia). In the Pittsburgh SMSA, by contrast, none of the 10 are in the City of Pittsburgh.

Tables 2a and 2b show regression parameters and Ginis for the three SMSAs. Table 2a presents coefficients for the elderly population, regardless of income. The coefficient for Baltimore is 0.4799, approximately the same as Philadelphia (0.4868). Both are substantially higher than Pittsburgh (0.3890).[2] This confirms earlier conjectures about Pittsburgh's elderly population being more dispersed throughout the metropolitan area.

Table 2b shows estimates for the poor elderly population. As hypothesised, they are more concentrated than the elderly population alone. The coefficients for Baltimore and Philadelphia are 0.7765 and 0.7553. Both are considerably higher than Pittsburgh's 0.5804. The poor elderly population in Baltimore is marginally more concentrated than Philadelphia and both are considerably more concentrated than Pittsburgh.

A summary measure (RATIO at the bottom of Table 2b) can be provided by dividing the Gini for the *poor* elderly by that for *all* elderly. If the poor elderly are more (less) concentrated than the elderly, then RATIO should be greater (less) than one. In Baltimore, Philadelphia and Pittsburgh it equals 1.618, 1.552, and 1.492, respectively. Pittsburgh, then, has a less concentrated elderly population than the other two metropolitan areas; its poor elderly are also less concentrated relative to all elderly.

Table 2a

Elderly Gini estimates

	Baltimore	Philadelphia	Pittsburgh
Constant	−1.0574	−1.0414	−1.2785
	(.0672)	(.0651)	(.0779)
α	.8806	.8852	.8569
	(.0249)	(.0241)	(.0289)
β	.4351	.4354	.4312
	(.0117)	(.0113)	(.0136)
a	.3466	.3522	.2778
R^2	.9786	.9800	.9702
Std error	.4122	.3993	.4807
GINI	.4799	.4868	.3890

Table 2b

Poor-elderly estimates

	Baltimore	Philadelphia	Pittsburgh
Constant	−.5505	−.5792	−.8581
	(.0597)	(.0593)	(.0671)
α	.9304	.9289	.8960
	(.0218)	(.0217)	(.0247)
β	.4474	.4467	.4403
	(.0101)	(.0101)	(.0116)
a	.5756	.5594	.4230
R^2	.9850	.9851	.9796
Std error	.3575	.3560	.4080
GINI	.7765	.7553	.5804
RATIO	1.618	1.552	1.492

Conclusions

This paper has proposed Lorenz curves, Gini coefficients, and ratios of the Ginis as summary measures for the spatial distribution of elderly populations in

[2] Explicit confidence intervals for G are not readily calculated. The values of the regression coefficients (particularly a) are significantly smaller for Pittsburgh than for Baltimore or Philadelphia.

metropolitan areas, using Baltimore, Philadelphia and Pittsburgh as specific examples. The elderly are more concentrated in the central cities, particularly in Philadelphia and Baltimore. The Pittsburgh SMSA has a less concentrated elderly population than do the other two, even though its overall elderly percentage is higher.

It is difficult to generalise in a sample of only three metropolitan areas. A more detailed study would examine a larger sample with concentration measures related to metropolitan age and growth, housing opportunities, and in- and outmigration. Still further work might relate the concentration measures to the costs of providing various elderly-specific services.

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