3

Monotone Censored Data

3.1 Data Structure and Model

Let $\{X(t) : t \in \mathbb{R}_{\geq 0}\}$ be a multivariate stochastic process indexed by time t. Let T denote an endpoint of this stochastic process, and define $X(t) \equiv X(\min(t,T))$. Let $R(t) = I(T \leq t)$ be one of the components of $X(t)$. We define the full data as $X = \bar{X}(T) = (X(s) : s \leq T)$, where T is thus a function X.

Suppose that we observe the full data process $X(\cdot)$ up to the minimum of a univariate censoring variable C and T so that for the observed data we have:

$$Y = (\tilde{T} = \min(T,C), \Delta = I(T \leq \tilde{T}) = I(C \geq T), \bar{X}(\tilde{T})).$$

We will define $C = \infty$ if $C > T$ so that this data structure can be represented as

$$Y = (C, \bar{X}(C)).$$

In the next section, we provide several important examples of this monotone censored data structure.

Let $\mathcal{M}^F$ be a specified full data model for the distribution F_X of X, and let $\mu = \mu(F_X) \in \mathbb{R}^k$ be the full data parameter of interest. Let $G(\cdot \mid X)$ be the conditional distribution of C, given X, and it is assumed that G satisfies CAR (i.e., $G \in \mathcal{G}(CAR)$). Given working models $\mathcal{M}^{F,w} \subset \mathcal{M}^F$ and $\mathcal{G} \subset \mathcal{G}(CAR)$, we define the observed data model $\mathcal{M} = \{P_{F_X,G} : F_X \in \mathcal{M}^{F,w}\} \cup \{P_{F_X,G} : G \in \mathcal{G}\}$. We also define the observed data model

$\mathcal{M}(\mathcal{G}) = \{P_{F_X,G} : F_X \in \mathcal{M}^F, G \in \mathcal{G}\}$. Candidates for the censoring model $\mathcal{G}$ are given below.

Let $g(c \mid X)$ be the conditional density of C, given X, either w.r.t. a Lebesgue density or counting measure, and let $\lambda_C(c \mid X)$ be the corresponding conditional hazard. Define $A(t) \equiv I(C \leq t)$. The conditional distribution G satisfies CAR if

$$E(dA(t) \mid X, \bar{A}(t-)) = E(dA(t) \mid \bar{A}(t-), \bar{X}(\min(t,C))).$$

In other words, the intensity of $A(t)$ w.r.t. the unobserved history $(X, \bar{A}(t-))$ should equal the intensity of $A(t)$ w.r.t the observed history $(\bar{A}(t-), \bar{X}(\min(t,C)))$. Equivalently, G satisfies CAR if for $c < T$

$$\lambda_C(c \mid X) = m(c, \bar{X}(c)) \text{ for some measurable function } m. \quad (3.1)$$

If C is continuous, then a practical and useful submodel $\mathcal{G} \subset \mathcal{G}(CAR)$ (3.1) is the multiplicative intensity model w.r.t. the Lebesgue measure

$$E(dA(t) \mid \bar{A}(t-), \bar{X}(\min(t,C))) = I(\tilde{T} > t)\lambda_0(t)\exp\left(\alpha_0^\top W(t)\right),$$

where α_0 is a k-dimensional vector of coefficients, $W(t)$ is a k-dimensional time-dependent vector that is a function of $\bar{X}(t)$, and λ_0 is an unspecified baseline hazard. Note that

$$\lambda_C(t \mid X, T > t) \equiv \lambda_0(t)\exp\left(\alpha_0^\top W(t)\right)$$

denotes the Cox proportional hazards model for the conditional hazard λ_C.

If we knew that the censoring was independent of the survival time and the history, then, for $t < T$, this would reduce to

$$\lambda_C(t \mid X) = \lambda_0(t).$$

If C is discrete, then a natural model $\mathcal{G} \subset \mathcal{G}(CAR)$ is

$$E(dA(t) \mid \bar{A}(t-), \bar{X}(\min(t,C))) = I(\tilde{T} \geq t)\frac{1}{1+\exp(-\{h_0(c) + \alpha_0^\top W(t)\})},$$

where h_0 could be left unspecified. This corresponds with assuming a logistic regression model for the conditional censoring hazard $\lambda_C(t \mid X) = P(C = t \mid X, C \geq t)$: for $t < T$

$$\log\left(\frac{\lambda_C(t \mid X)}{1-\lambda_C(t \mid X)}\right) = h_0(t) + \alpha_0^\top W(t).$$

If the support of C gets finer and finer so that $P(C = t \mid X, C \geq t)$ approximates zero, then this model with h_0 unspecified converges to the Cox proportional hazards model with $\lambda_0 = \exp(h_0)$ and regression coefficients α_0 (see e.g., Kalbfleish and Prentice, 1980).

Whatever CAR model for $\lambda_C(t \mid X)$ is used, the G part of the density of $P_{F_X,G}$ in terms of $\lambda_C(t \mid X)$ is given by the partial likelihood of $A(t) = I(C \leq t)$ w.r.t. history $\mathcal{F}(t) \equiv (\bar{A}(t-), \bar{X}(\min(t,C)))$ as defined in Andersen, Borgan, Gill and Keiding (1993) for the continuous case. Let