

$\mathbf{F}_X$ : the probability distribution of the full data  $X$ .

$\mathbf{G}$ : the conditional probability distribution of  $C$ , given  $X$ , also called the censoring mechanism, treatment mechanism, or action mechanism, depending on what  $C$  stands for. When we define CAR, we do this in terms of the conditional distribution of  $Y$ , given  $X$ , which is determined by  $G$ , and, by CAR, it is the identifiable part of  $G$ . In this book we frequently denote the conditional distribution of  $Y$ , given  $X$ , with  $G$  as well.

$\mathbf{P}_{\mathbf{F}_X, \mathbf{G}}$ : the distribution of the observed data  $Y$ , which only depends on  $G$  through the conditional distribution of  $Y$ , given  $X$ .

$\mathcal{G}$ : a model for the censoring mechanism  $G$  (i.e., it is known that  $G \in \mathcal{G}$ ).

$\mathcal{G}(\text{CAR})$ : all conditional distributions  $G$  of  $C$ , given  $X$ , satisfying censoring at random (CAR).

$\mathcal{M}^F$ : a model for  $F_X$  (i.e., the full data model).

$\mathcal{M} = \{P_{F_X, G} : F_X \in \mathcal{M}^{F, w}, G \in \mathcal{G}(\text{CAR})\} \cup \{P_{F_X, G} : F_X \in \mathcal{M}^F, G \in \mathcal{G}\}$ , the observed data model allowing that either the working model  $\mathcal{M}^{F, w}$  for  $F_X$  or the censoring model  $\mathcal{G}$  for  $G$  is misspecified, but not both.

$\mathcal{M}(G) = \{P_{F_X, G} : F_X \in \mathcal{M}^F, G \in \mathcal{G}\}$ : the observed data model when assuming a correctly specified model  $\mathcal{G}$  for  $G$ .

$\mathcal{M}(\mathbf{G}) = \{P_{F_X, G} : F_X \in \mathcal{M}^F\}$ : the observed data model if the censoring mechanism  $G$  is known.

$\mathcal{M}(\text{CAR}) = \{P_{F_X, G} : F_X \in \mathcal{M}^F, G \in \mathcal{G}(\text{CAR})\}$ : the observed data model if the censoring mechanism is only known to satisfy CAR.

$\mu = \mu(\mathbf{F}_X) \in \mathbb{R}^k$ : the Euclidean parameter of  $F_X$  of interest.

$\mathbf{Z} = \mathbf{g}(X^* | \alpha) + \epsilon$ : a multivariate generalized regression model (a particular choice of full data model), where  $Z$  is a  $p$ -variate outcome,  $X^*$  is a vector of covariates,  $g(X^* | \alpha)$  is a  $p$ -dimensional vector whose components are regression curves parametrized with a regression parameter  $\alpha \in \mathbb{R}^k$ ,  $\epsilon$  is a  $p$ -variate residual satisfying  $E(K(\epsilon_j) | X^*) = 0$ ,  $j = 1, \dots, p$ , for a given monotone nondecreasing function  $K$ .

$\mathbf{K}(\cdot)$ : monotone function specifying the location parameter (e.g., mean, median, truncated mean, smooth median) of the conditional distribution of  $Z$ , given  $X^*$ , modeled by  $g(X^* | \alpha)$ . For example, 1)  $K(\epsilon) = \epsilon$ , 2)  $K(\epsilon) = I(\epsilon > 0) - (1 - p)$ , 3)  $K(\epsilon) = \epsilon$  on  $[-\tau, \tau]$  and  $K(\epsilon) = \tau$  for  $\epsilon > \tau$ ,  $K(\epsilon) = -\tau$  for  $\epsilon < -\tau$  correspond with mean regression,  $p$ th quantile regression (e.g.,  $p = 0.5$  gives median regression), and truncated mean regression, respectively.

$\mathbf{N}(t)$ : a counting process being a part of the full data  $X$ .

$\lambda(t)dt = E(dN(t) | \bar{Z}(t-)) = Y(t)\lambda_0(t)\exp(\beta W(t))dt$ : a multiplicative intensity model (a particular full data model), where  $\bar{Z}(t)$  is a given function of  $\bar{X}(t)$  including the past  $\bar{N}(t)$  of the counting process  $N$ ,  $Y(t)$  is an indicator function of  $\bar{Z}(t-)$  (indicator that  $N(\cdot)$  is at risk of jumping at time  $t$ ), and  $W(t)$  is a vector of covariates extracted from  $\bar{Z}(t-)$ . We also consider the case where  $\bar{Z}(t-)$  does not include the past of  $N$ . In this case, we refer to these models as proportional rate models. We also consider discrete multiplicative intensity models, where  $\lambda(t) = Y(t)\Lambda_0(dt)\exp(\beta W(t))$  is

now a conditional probability.

$\bar{\mathbf{F}} = \mathbf{1} - \mathbf{F}$ .

$\langle \mathbf{S}_1, \dots, \mathbf{S}_k \rangle$ : linear span of  $k$  elements (typically scores in  $L_0^2(F_X)$  or  $L_0^2(P_{F_X, G})$ ) in a Hilbert space.

$\langle \bar{\mathbf{S}} \rangle \equiv \langle \mathbf{S}_1, \dots, \mathbf{S}_k \rangle$ : linear span of the  $k$  components of  $\bar{\mathbf{S}}$ .

$\langle \mathbf{f}, \mathbf{g} \rangle$ : inner product defined in a Hilbert Space.

$\langle \mathbf{f}, (\mathbf{g}_1, \dots, \mathbf{g}_k) \rangle \equiv (\langle \mathbf{f}, \mathbf{g}_1 \rangle, \dots, \langle \mathbf{f}, \mathbf{g}_k \rangle)$ .

$\mathbf{H}_1 \oplus \mathbf{H}_2 = \{h_1 + h_2 : h_j \in H_j, j = 1, 2\}$ : the sum space spanned by two orthogonal sub-Hilbert spaces  $H_1, H_2$  of a certain Hilbert space.

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$\Pi(\cdot | \mathbf{H})$ : the projection operator onto a subspace  $H$  of a certain Hilbert space.

$L_0^2(\mathbf{F}_X)$ : Hilbert space of functions  $h(X)$  with  $E_{F_X}h(X) = 0$  with inner product  $\langle h, g \rangle_{F_X} = E_{F_X}h(X)g(X)$  and corresponding norm  $\|h\|_{F_X} = \sqrt{E_{F_X}h^2(X)}$ .

$T^F(\mathbf{F}_X) \subset L_0^2(F_X)$ : the tangent space at  $F_X$  in the full data model  $\mathcal{M}^F$ . This is the closure of the linear space spanned by scores of a given class of one-dimensional submodels  $\epsilon \rightarrow F_\epsilon$  that cross  $F_X$  at  $\epsilon = 0$ .

$T_{\text{nuis}}^F(\mathbf{F}_X) \subset L_0^2(F_X)$ : the nuisance tangent space at  $F_X$  in the full data model  $\mathcal{M}^F$ . This is the closure of the linear space spanned by scores of a given class of one-dimensional submodels  $\epsilon \rightarrow F_\epsilon$  that cross  $F_X$  at  $\epsilon = 0$  and satisfy  $d/d\epsilon\mu(F_\epsilon)|_{\epsilon=0} = 0$ .

$T_{\text{nuis}}^{F, \perp}(\mathbf{F}_X) \subset L_0^2(F_X)$ : the orthogonal complement of the nuisance tangent space  $T_{\text{nuis}}^F(F_X)$  in model  $\mathcal{M}^F$ , where  $\mu$  is the parameter of interest. The class of full data estimating functions  $D_h(\cdot | \mu, \rho)$ ,  $h \in \mathcal{H}^F$ , is chosen so that  $T_{\text{nuis}}^{F, \perp}(F_X) \supset \{D_h(X | \mu(F_X), \rho(F_X)) : h \in \mathcal{H}^F\}$ , where the right hand side is chosen as rich as possible so that we might even have equality.

$\mathbf{D}_h$ , defined in full data model  $\mathcal{M}^F$ : full data estimating function  $D_h : \mathcal{X} \times \{(\mu(F_X), \rho(F_X)) : F_X \in \mathcal{M}^F\} \rightarrow \mathbb{R}$  for parameter  $\mu$  with nuisance parameter  $\rho$ . Here  $h \in \mathcal{H}^F$  indexes different possible choices of full data estimating functions.

$\mathcal{H}^F$ : index set providing a rich class of full data estimating functions satisfying:

$$D_h(X | \mu(F_X), \rho(F_X)) \in T_{\text{nuis}}^{F, \perp}(F_X) \text{ for all } h \in \mathcal{H}^F.$$

$$D_h, h \in \mathcal{H}^{F, k}: \text{ for } h = (h_1, \dots, h_k) \in \mathcal{H}^{F, k},$$

$$D_{h_1, \dots, h_k}(X | \mu, \rho) = (D_{h_1}(X | \mu, \rho), \dots, D_{h_k}(X | \mu, \rho)).$$

A full data structure estimating function  $D_h$ ,  $h \in \mathcal{H}^{F, k}$ , defines an estimating equation for  $\mu$ : given an estimate of  $\rho$ , one can estimate  $\mu$  with the solution of the  $k$ -dimensional equation  $0 = \sum_{i=1}^k D_h(X_i | \mu, \rho_n)$ .

$\mathbf{D}_h(\mathbf{X} | \mu, \rho)$ : full data estimating function  $D_h$  evaluated at  $X, \mu, \rho$ . Sometimes,  $D_h(X | \mu, \rho)$  is used to denote the actual estimating function,